Aiming for the Bull's Eye: Inflation Targeting under Uncertainty*

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April 2009

Abstract

We study the implications of uncertainty for inflation targeting. We apply Brainard's static framework which assumes multiplicative uncertainty in the monetary transmission. Brainard's main result is that in the presence of uncertainty, monetary authorities become naturally more cautious. But this also implies that monetary objectives are seldom achieved. We therefore attempt to find a monetary rule that reaches the objectives set more often and improves the welfare of the Central Bank. Such a rule is the result of a new algorithm that we put forward, in which the inflation target is state contingent. The Central Bank sets therefore (as an auxiliary step), a variable inflation target that depends on both the degree of uncertainty as well as the shocks that occur each time. If the benefits of reaching the inflation target are properly accounted for in the loss function, we show that such an optimisation procedure helps the CB attain its objectives more often, thereby reducing the losses incurred. Moreover, and as a corollary to such an approach, the rule derived is *ex ante* neutral to the degree of uncertainty.

J.E.L. Classification: E42, E52
Keywords: Inflation Targeting, Brainard Uncertainty, Two-Step Target, Credibility

*Views expressed are our own and do not necessarily reflect those of the DNB. We are grateful to Peter van Els, Lex Hoogduin, Andy Hughes Hallett, Willem Verhagen, Paul De Grauwe, Athanasios Orphanides, Jaques Melitz, Alan Sutherland and seminar participants at the DNB, the Universities of Amsterdam and Strathclyde, Boston College, EBRD, Bank of England, ECB, and the 6th International Conference on Macroeconomic Analysis at the University of Crete, for comments and suggestions. All remaining errors are naturally ours.

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1 Introduction

The benefits of inflation targeting in the Svensson (1999) sense amount to providing a nominal anchor for the private sector to infer policies with, in order to formulate expectations with greater accuracy[1]. For the Central Bank (CB) on the other hand, inflation targeting provides an implicit commitment mechanism which increases its cost of deviating from announced targets and hence discourages it from doing so. The economy on the whole benefits from greater transparency because it leads to greater credibility and by consequence to effective monetary policies. From a political economy standpoint therefore, the literature associates the concept of inflation targeting with greater transparency and hence with more credible and effective policies. By the same token, a central bank that fails to achieve the target that it sets (and announces) will be penalised with a loss in credibility and hence a subsequent reduction in the ability to pursue its objectives. "It appears that for monetary policy makers, announcements alone are not enough; the only way to gain credibility is to earn it", (Bernanke and Mishkin, 1997).

In this paper we analyse the effects of inflation targeting in an economy characterised by parameter uncertainty, as modelled by Brainard, (1967). In a very simple static framework similar to the Brainard framework itself, we observe that the attenuation effect put forward by Brainard implies a failure to attain the preannounced inflation target, on average. This is caused by the fact that the private sector, in full knowledge of the extent of the uncertainty that prevails, discounts the ability of the Central Bank to achieve its objectives and forms expectations which are different to the target. As a result of these observations, we analyse two issues: *first*, if there is some value in attaining the target, then we aim to find an algorithm that will both achieve it on average, as well as still operate in an optimisation framework, such that the procedure remains transparent to the public. We will thus identify a two-step algorithm. In the first step, the central bank deviates from the target in order to reactivate the instrument and only in the second does it aim for the actual target itself. The two-step procedure amounts therefore, to the Central bank aiming *for* the bull's eye, and not directly *at* it. *Second*, we identify the conditions of uncertainty under which such an algorithm can prove superior to the Brainard result. This requires redefining the framework somewhat, in order to appropriately reward (penalise) the central bank, if it achieves (misses) the target. This is important in an inflation targeting framework as announcing a target that is unlikely to be achieved is not necessarily increasing one's credibility (Posen, 2002). We argue that the standard framework for analysing welfare losses falls short of evaluating an inflation targeting regime. Our attempt will allow for an explicit role for private sector expectations and hence quantify the losses that the Central Bank suffers if it misses its inflation target. We will identify this with the '*loss in credibility*' that the monetary authority incurs.

The paper is organised as follows. Section 2 describes the model used and presents our two benchmark cases: first, that of Certainty in the parameters and second the Brainard result from his 1967 seminal paper with multiplicative uncertainty. We extend this analysis in section 3, by introducing a two-step inflation targeting

[1] See Levin et al (2004), Gürkaynak et al (2006) and Demertzis and Viegi (2008).

procedure and describing the algorithm in detail. This constitutes the main contribution of the paper. Section 4 evaluates the benefits of such an algorithm in a framework that allows explicitly for the benefits of inflation targeting and section 5 demonstrates these results with the aid of numerical simulations. Section 6 concludes.

2 The Model

In order to keep the comparison with Brainard (1967) feasible, consider an economy described by a simple reduced form of a demand-supply static system as follows:

$$\pi = -ai + \varepsilon \tag{1}$$
$$y = \pi - \pi^e + \eta \tag{2}$$

where (1) represents a demand equation, in which deviations of inflation from a given starting point are a function of i, (denoting the policy makers' intended deviation of their instrument from its neutral level) and (2) is a traditional expectations augmented Phillips curve[2]. Term a can be either a constant (and positive) parameter if we assume certainty or it may be stochastic in nature, drawn from a normal distribution $a \rightarrow N\left(\overline{a}, \sigma_a^2\right)$, in line with Brainard's methodology. Terms ε and η represent a demand and supply shock respectively[3] and are independently normally distributed variables with known properties, $\varepsilon \rightarrow N(0, \sigma_\varepsilon^2)$ and $\eta \rightarrow N(0, \sigma_\eta^2)$. We consider a static but sequential game between the Central Bank and the private sector. The latter forms expectations at the start of the period about the level of inflation at the end of the period. These expectations form the basis which to base wage negotiations on, such that $w = \pi^e$. A shock occurs next and the CB reacts by choosing that interest rate which optimises the conditional expectation of its loss function, expressed in terms of deviations of inflation and output from their

[2]Traditionally equation (1) is written as $\pi - \pi^* = -ai + \varepsilon$ where π^* is the level of inflation that the CB targets (see for example, Faust and Svensson, 2001 and Schellekens, 2002). But this implies that expectations are always tied to the announcements (i.e. $\pi^e = \pi^*$) and in the absence of shocks, monetary policy needs to set the interest rate equal to the natural rate (i.e. $i = 0$) for inflation to reach its target. Our task however, will be to show how uncertainty can prevent an announced target from being credible (and therefore $\pi^e \neq \pi^*$). Equation (1) is thus equivalent to $\pi - \pi_{-1} = -ai + \varepsilon$, where $\pi_{-1} = 0$ and $\pi^* \neq 0$; we will use this simplified version in the main text but Appendix A1 and B1 will provide derivations for the generalised case. In the Barro-Gordon (1983) set-up, equation (1) is consistent with a vertical long-run Phillips curve and a negatively sloped short-run curve which the Central Bank wants to shift to a different level of assumed expectations. Replacing (1) with $\pi - \pi^* = -ai + \varepsilon$ implies that expectations are now fixed and the central bank moves the instrument to deal with shocks, that make it move along a given short-run curve. In that respect, this traditional set-up is not suitable to our aim.

[3]ε is an implementation or control error. We assume for simplicity purposes that a, ε and η are all independent of each other.

respective targets.

$$\min_i E(L) = \frac{1}{2}E\left[(\pi - \pi^*)^2 + y^2\right] \tag{3}$$

Loss function (3) shows that the Central Bank follows a flexible inflation targeting rule, as defined by Svensson (1999). We have attached equal weights to the two objectives, for simplicity purposes. Furthermore, in the absence of any other policy agent in the economy, the Central Bank's objectives are identified with those of the median voter. At the end of the period, the effects of the Central Bank's policies are revealed and in a rational expectations world, the discretionary outcome occurs. There is symmetric information shared across the agents with respect to both the sequence of events as well as the existence of uncertainty. The only difference in information therefore, (given the timing of the game) is that private agents have no knowledge of the shock, whereas the CB reacts to it, in full knowledge of its extent.

Table 1 summarises the results produced under the assumption of Certainty (CE), as well as under transmission uncertainty *à la* Brainard (see appendix A and B for detailed derivations)[4].

Table 1: Equilibrium Values

Variable	Certainty	Brainard Uncertainty
i	$-\frac{1}{a}\pi^* + \frac{1}{2a}(2\varepsilon + \eta)$	$-\frac{\overline{a}}{\overline{a}^2+2\sigma_a^2}\pi^* + \frac{\overline{a}}{2\left(\overline{a}^2+\sigma_a^2\right)}(2\varepsilon + \eta)$
π^e	π^*	$\frac{\overline{a}^2}{\overline{a}^2+2\sigma_a^2}\pi^*$
π_{RE}	$\pi^* - \frac{1}{2}\eta$	$\frac{\overline{a}^2}{\overline{a}^2+2\sigma_a^2}\pi^* + \frac{2\sigma_a^2\varepsilon-\overline{a}^2\eta}{2\left(\overline{a}^2+\sigma_a^2\right)}$
y_{RE}	$\frac{1}{2}\eta$	$\frac{\sigma_a^2}{\overline{a}^2+\sigma_a^2}\varepsilon + \frac{\overline{a}^2+2\sigma_a^2}{2\left(\overline{a}^2+\sigma_a^2\right)}\eta$

As the table above demonstrates, increasing uncertainty confirms Brainard's observations on optimal monetary policy. In particular, the presence of uncertainty has the following effects:

- The use of the instrument is constrained. In Brainard's terminology, the policy maker becomes naturally more cautious and at the limit abandons it altogether.(i.e. $\lim_{\sigma_a^2 \to \infty} i = 0$) [5].

- The inflation target itself becomes less important in the formulation of expectations about future inflation. The private sector discounts the ability of the central bank to achieve the announced inflationary target, in proportion to the

[4]Other attempts which deal with a similar type of uncertainty include Ellison and Valla (2001), Onatski (2000), Sack (2000), Söderström (2002), among others.

[5]It is important to note that the Brainard attenuation effect depends on the source of uncertainty assumed. Brainard himself was aware that uncertainty some times calls for a more aggressive response depending on the covariances between the varying model parameters and the error terms. Furthermore, Craine (1979) shows that the timing is of crucial importance. Increases in future uncertainty thus raise the current level of the average policy response, whereas increases in current uncertainty lower it.

level of uncertainty. At the same time, this implies that on average, inflation will never reach its target.

- Both supply and demand shocks have an effect on output in the presence of uncertainty because monetary policy is unable to insulate the real side from demand shocks.

Comparing the two different scenarios, the presence of uncertainty reduces the efficiency of monetary policy making, which is thus moved away from its first best.

3 Inflation Targeting in Two Steps

The direct corollary of a less effective monetary policy is that the objectives of the Central Bank are also seldom achieved. What we investigate next is whether there exists a policy rule that can help monetary authorities achieve their assumed targets more often and thus reduce their welfare costs. We do so, by introducing an implicit target $(\pi^* + \theta)$[6], θ being the implicit component subject to discretionary change, still to be determined. It acquires therefore, the role of a choice variable and π^* assumes now the role of a *declaration of intent* or *deep* target. The timing of the game is described in figure (1) below.

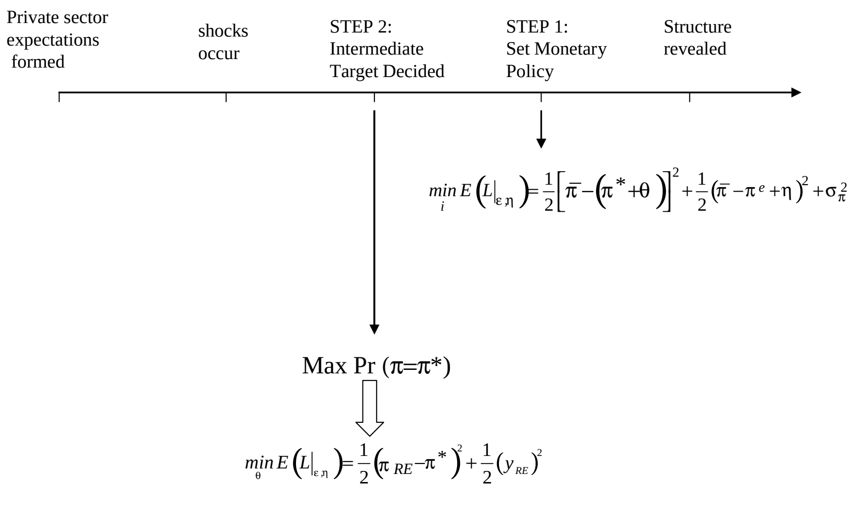

Figure 1: Timing of Events

The algorithm that we put forward implies therefore that the CB optimises its actions in two steps (solved backwards).

[6] Vredin and Warne 2000 argue that one could draw π^* from a distribution to reflect uncertainty in the CB's preferences. We do not allow for such asymmetry here and justify therefore any deviations from π^* as a means of dealing with uncertainty. Indeed, parameter θ will vary according to the level of uncertainty.

3.1 Step 1

The Central Bank identifies the optimal rule as a function of θ. In other words it optimises the expected value of the following auxiliary objective function:

$$\min_i E(L) = \frac{1}{2} E\left\{[\pi - (\pi^* + \theta)]^2 + y^2\right\} \tag{4}$$

or the conditional expectation, given (1) and (2)

$$\min_i E(L\mid_{\varepsilon,\eta}) = \frac{1}{2}\left\{[-\overline{a}i - (\pi^* + \theta) + \varepsilon]^2 + (-\overline{a}i - \pi^e + \varepsilon + \eta)^2\right\} + i^2\sigma_a^2 \tag{5}$$

following Brainard's methodology (see Appendix B). Optimising (5) gives the following monetary policy reaction function and resulting inflation, for given private sector expectations.

$$\begin{aligned}
i &= -\frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left[(\pi^* + \theta) + \pi^e\right] + \frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}(2\varepsilon + \eta) && (6)\\
E(\pi\mid_{\varepsilon,\eta}) &= \frac{\overline{a}^2}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left[(\pi^* + \theta) + \pi^e\right] + \frac{2\sigma_a^2\varepsilon - \overline{a}^2\eta}{2\left(\overline{a}^2 + \sigma_a^2\right)} && (7)
\end{aligned}$$

Based on (7), the private sector anticipates the following rate of inflation:

$$\pi^e = \frac{\overline{a}^2}{\overline{a}^2 + 2\sigma_a^2}(\pi^* + \bar{\theta}) \tag{8}$$

where $\bar{\theta}$ is the *ex ante* average departure from the target anticipated by the private sector. As we will show further down, $\bar{\theta}$ is always positive, a feature specific to our model since inflationary expectations achieved under Brainard uncertainty fall always short of π^*[7]. The respective Rational Expectations solutions are then:

$$\begin{aligned}
i_{RE} &= -\frac{\overline{a}}{\overline{a}^2 + 2\sigma_a^2}\left[\pi^* + \frac{\overline{a}^2}{2\left(\overline{a}^2 + \sigma_a^2\right)}\bar{\theta}\right] + \frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}(2\varepsilon + \eta - \theta) && (9)\\
\pi_{RE} &= \frac{2\overline{a}^2\left(\sigma_a^2 + \overline{a}^2\right)\pi^* + 4\varepsilon\sigma_a^2 + 2\overline{a}^2\sigma_a^2(\varepsilon - \eta + \theta) + \overline{a}^4\left(\theta + \bar{\theta} - \eta\right)}{2\left(\overline{a}^2 + \sigma_a^2\right)\left(\overline{a}^2 + 2\sigma_a^2\right)} && (10)\\
y_{RE} &= \frac{\sigma_a^2}{\left(\overline{a}^2 + \sigma_a^2\right)}\varepsilon + \frac{\left(\overline{a}^2 + 2\sigma_a^2\right)}{2\left(\overline{a}^2 + \sigma_a^2\right)}\eta + \frac{\overline{a}^2(\theta - \bar{\theta})}{2\left(\overline{a}^2 + \sigma_a^2\right)} && (11)
\end{aligned}$$

The above three equation rules imply that for a given level of uncertainty, the CB will choose to deviate from its ultimate target π^* by a given θ (and on average by $\bar{\theta}$).

[7] This is because inflation is zero to start with, $(\pi_{-1} = 0)$.The Central Bank needs therefore to take some action in order to get to π^*, even in the absence of shocks.

3.2 Step 2

But the degree of deviation θ is chosen optimally. In other words, the CB applies θ to maximise the probability of achieving its true objectives. The derived rules from Step 1 for π, (10) and y, (11) are thus substituted into the objective function of the Central Bank:

$$\min_{\theta} E(L\mid_{\varepsilon,\eta}) = \frac{1}{2}E\left[(\pi_{RE}-\pi^*)^2 + y_{RE}^2\right] \tag{12}$$

to produce[8]

$$\min_{\theta} E(L\mid_{\varepsilon,\eta}) = f(\theta,\bar{\theta},\sigma_a^2,\varepsilon,\eta) \tag{13}$$

Given the rules, the aim of the CB is to find the optimal deviation θ, contingent on the shock hitting the economy and the perceived uncertainty of the transmission of policies, that will get her closer to π^*. In other words,

$$\theta(\sigma_a^2,\varepsilon,\eta) = \arg\min_{\theta} E(L\mid_{\varepsilon,\eta})$$

which in its analytical form is

$$\theta = \frac{\sigma_a^2\left[-2\sigma_a^2(2\varepsilon+\eta-\pi^*) + \overline{a}^2(-2\varepsilon-\eta+\bar{\theta}+2\pi^*)\right]}{(\overline{a}^4+2\overline{a}^2\sigma_a^2)} \tag{14}$$

$$E(\theta) = \frac{2\sigma_a^2\pi^*}{\overline{a}^2} \tag{15}$$

For $\pi^* > 0$, we have that $E(\theta)$ (or $\bar{\theta}$) > 0, such that $\pi^e \to \pi^*$. Substituting $E(\theta)$ into (14) gives a solution for θ:

$$\theta = -\frac{\sigma_a^2\,(2\varepsilon+\eta-2\pi^*)}{\overline{a}^2} \tag{16}$$

As uncertainty decreases, the deviations from π^* decrease as well, such that at the limit they become zero, i.e.

$$\lim_{\sigma_a^2\to 0}(\theta) = 0$$

Proposition 1 *Applying a two-step procedure in which θ is contingent on the shocks that hit the economy, the existing uncertainty and the inflation target, neutralises the ex ante effects of uncertainty on the policy rules.*

[8] Note that in (12), $E(L\mid_{\varepsilon,\eta}) = \frac{1}{2}\left[(\pi_{RE}-\pi^*)^2 + y_{RE}^2\right]$ as there are no random variables.

Proof 1: Substituting the analytical solutions for θ, (16) and $E(\theta)$, (15) into (8) - (11) produces the *two-step* target rules that a Central Bank needs to apply under uncertainty.

$$\begin{aligned} \pi^e &= \pi^* && (17) \\ i_{RE} &= -\frac{1}{\bar{a}}\pi^* + \frac{1}{2\bar{a}}(2\varepsilon + \eta) && (18) \\ \pi_{RE} &= \pi^* - \frac{1}{2}\eta && (19) \\ y_{RE} &= \frac{1}{2}\eta && (20) \end{aligned}$$

The rules achieved are similar to those attained under CE (with a replaced by $\bar{a}$). This demonstrates that by varying the target optimally, uncertainty in the transmission process is neutralised. This result is a direct consequence of the way the inflation target becomes contingent on the shock incurred and the level of uncertainty. Parameter θ is then chosen to maximise the probability of hitting the explicit target π^*(or perhaps more accurately, a prespecified area around it). *Ex ante* therefore, we achieve a comparable result to that derived under full certainty[9].

4 Measuring the 'loss' in credibility

We turn next to the levels of welfare achieved by applying these two alternatively rules. Naturally, as Brainard's procedure (see table 1 for the exact formulations) derives the *optimal* rule that minimises (3), the *two-step* inflation target procedure (equations 17-20) will not produce superior welfare, on average. This can be seen by comparing $E(L_{BR})$ to $E(L_{TS})$, the losses evaluated for each of the two cases respectively. We therefore, have:

$$\begin{aligned} E(L_{BR}) &= \frac{V^2}{1+2V^2}{\pi^*}^2 && (21) \\ E(L_{TS}) &= V^2{\pi^*}^2 && (22) \end{aligned}$$

where $V = \frac{\sigma_a}{\bar{a}}$, the coefficient of variation ($C.V.$). It is straightforward to show that:

$$E(L_{BR}) < E(L_{TS}), \qquad \forall \quad V^2 > 0$$

The analysis so far however, points to an inconsistency between the way inflation targeting is discussed in the literature and the way it is actually modelled. Our motivation for looking for an alternative rule to that provided by Brainard, stemmed

[9] Our approach is in fact equivalent to introducing an extra instrument while the number of targets remains the same. As Hughes Hallett (1989) mentions "...all the instruments will be needed to combat uncertainty even when there are only a few targets compared to the number of instruments".

from the fact that private sector expectations differed from the actual inflation target announced, as seen in table 1. But nowhere in the framework used to analyse its merits, is such a deviation penalised. Similarly, the benefits of the *two-step* procedure in hitting the target are not properly rewarded. This is in our view an inconsistency, as inflation targeting is praised for its ability to tie down expectations to the preannounced target. By implication, the credibility sustained (lost) from hitting (missing) the target ought to be part of the Central Bank's loss function.

In an effort to demonstrate the relevance of this point, we will apply an alternative loss function, in which we include the price paid by the Central Bank for failing to persuade the public that it will hit the target. This would imply a loss in credibility, which is in line with our interpretation of the benefits of inflation targeting. We assume thus that the Central Bank's objective function allows now for the addition of a new argument, ω, such that

$$\min_{i} E(L) = \frac{1}{2}E\left[(\pi - \pi^*)^2 + y^2 + \omega\right] \tag{23}$$

where[10] $\omega = f(x)$ and $x = \pi^e - \pi^*$, and represents the penalty paid in loss terms, for not being credible[11]. We assume the following two properties hold:

$$\begin{array}{l} \text{a)} \quad f(0) = 0, \quad f(x) > 0, \quad \forall \;\; x \neq 0 \quad \text{and} \\ \text{b) if} \quad |x + \nu| > |x| \quad \text{then} \quad f(x + \nu) > f(x), \quad \forall \;\; \nu \neq 0 \end{array}$$

One could interpret ω along similar lines to a Walsh contract (Walsh, 95), except it is the private sector, and not the government, that this time penalises the Central Bank. This is not necessarily a concept that could be legislated as part of the performance objectives of the CB à la Walsh, but could be derived from first principles, starting with individuals' utility (who collectively form the private sector). Parameter ω thus encapsulates the '*price*' paid by the Central Bank when the private sector forms expectations that are different to the target set, and unlike a Walsh contract is not constrained to be linear.

Proposition 2 *For any positive loss in credibility, i.e.* $\omega > 0$*, there exists a level of uncertainty* V^2 *(or* σ_a^2 *for given* $\bar{a}$*), for which* $E(L_{TS}) < E(L_{BR})$*.*

Proof 2: Note that as ω is not a function of the instruments, its inclusion in the objective function will not alter the optimal rules derived for either of the two procedures. It is immediately obvious that ω is equal to zero for the TS rule (from 17) but positive for the BR rule (from table 1). This implies that $E(L_{BR})$ are

[10] This is similar to the objective function under an inflation forecast targeting rule (see objective function (5.10) in Svensson, 2003), but for one important difference. Term ω is a function of $\pi^e - \pi^*$, whereas Svensson uses $\pi_{t+1,t} - \pi^*$. The difference between π^e and $\pi_{t+1,t}$ is that the former is the private sector expectation of inflation allowing for the interest rate rule, whereas the latter is the central bank's forecast of inflation.

[11] Credibility is thus defined as the extent to which expectations are anchored to the target (see Bomfin and Rudebusch, 2000, Demertzis and Viegi, 2008).

now burdened by the deviation $f\left(\pi^e - \pi^*\right)$, which in itself depends on the degree of uncertainty that prevails. Respective losses therefore, are

$$\begin{aligned} E(L_{BR}) &= \frac{V^2}{1+2V^2}\pi^{*^2} + \frac{\omega}{2} \\ E(L_{TS}) &= V^2\pi^{*^2} \end{aligned}$$

and we can identify the conditions required for $E(L_{TS}) < E(L_{BR})$ to be satisfied. For any given ω, we thus require a V^2 such that:

$$V^2\pi^{*^2} < \frac{V^2}{1+2V^2}\pi^{*^2} + \frac{\omega}{2} \tag{24}$$

Condition (24) is true, if

$$V^2 < \frac{\omega + \left[\omega\left(\omega + 4\pi^{*^2}\right)\right]^{\frac{1}{2}}}{4\pi^{*^2}} \tag{25}$$

and the magnitudes on either side of the inequality are strictly positive. Thus, when the level of uncertainty (V) is smaller than what indicated by (25), then the TS rule can do better in welfare terms than Brainard's prescription.

5 Numerical Simulations

We illustrate next the welfare implications of the two alternative procedures through Monte Carlo simulations of the system of equations (1) and (2). Table 2 presents the results of 100,000 stochastic simulations for which we draw a random shock ε and parameter a, from $N(0,1)$ and $N(1, 0.5^2)$, respectively. We choose these specific values for the moments of a, in order to have a sufficiently small coefficient of variation which in turn reduces the likelihood of negative values for a[12]. The inflation target π^* is assumed to be 2 and the output gap target, 0. The first three columns show the results under fixed inflation targeting, *à la* Brainard and the last three, those under the *two-step* inflation targeting procedure, for i, π and y respectively.

Table 2 : Monte Carlo Simulations

	i_{BR}	π_{BR}	y_{BR}	i_{TS}	π_{TS}	y_{TS}
Mean	−1.33	1.336	0.003	−2.00	2.004	0.004
St. Dev.	0.798	0.801	0.801	0.992	1.117	1.117
Max.	2.035	6.171	4.837	2.211	9.201	7.209
Min.	−4.69	−5.47	−6.80	−6.19	−6.24	−8.24

[12] Negative values for a imply a perverse monetary policy effect. For the assumed coefficient of variation, $Pr(a < 0)$ is less than 3 *per cent*.

Table 2 shows that following a *two-step* inflation target brings the authority much closer to its objectives but at the cost of greater variability. As the Brainard Rule is actually the *optimal* rule that minimises (3), the *two-step* inflation target procedure will not produce superior welfare, on average. Table 3 demonstrates this by showing the relation between the coefficient of variation (CV) and average losses for the two procedures. However, table 3 also shows the frequency with which the *two-step* inflation target becomes welfare improving (last column). It is interesting thus to note that for low levels of uncertainty, the welfare losses are very similar between the two rules (i.e. the superiority of the Brainard rule is less relevant) as well as that the frequency with which the *two-step* procedure is actually doing better, becomes greater than 50 *per cent*.

Table 3 : Analysis of Losses based on (3)

C.V.	$E(L_{BR})$	$E(L_{TS})$	$\%(L_{TS} < L_{BR})$
1	1.83	4.98	46%
0.5	0.86	1.25	56%
0.25	0.28	0.31	60%

Table 4 then shows the losses incurred when one considers the alternative loss function, (23). We assume for simplicity a linear functional form for credibility, i.e. $\omega = |\pi^e - \pi^*|$ in order to impose a relatively mild penalty on missing the target. The superiority of the Brainard rule is no longer unequivocal, as the inability to tie down expectations to the preannounced target is penalised by the private sector. For lower levels of uncertainty, the TS procedure will do better than Brainard's caution (with 73 and 83 *per cent* frequencies) and it will even do better on average (for $CV = 0.25$). This therefore, reintroduces the trade-off between the benefits of hitting the target on the one hand, and the costs of introducing variability in the system through the use of the instrument, on the other.

Table 4 : Analysis of Losses based on (23)

C.V.	$E(L_{BR})$	$E(L_{TS})$	$\%(L_{TS} < L_{BR})$
1	2.50	5.00	64%
0.5	1.20	1.24	73%
0.25	**0.39**	**0.31**	83%

6 Conclusions

Performing monetary policy under uncertainty presents the policy maker with the following trade-off: on the one hand, there is a need to be transparent and credible in order to tie down private sector expectations to the preferred inflation path; on the other hand, there is a need to take account of the risk implied by the stochastic nature of economy. While inflation targeting gives an answer to the first objective, there is a need to test its robustness under conditions of uncertainty. The traditional framework for analysing monetary policy under uncertainty based on Brainard (1969), concludes that it is optimal to use the instrument with caution, internalizing the imperfect knowledge of the economy in the policy making. We show that in an inflation targeting set-up this means that the target is on average missed. By implication, one of the main objectives of an inflation targeting regime is lost, since the private sector discounts the objectives of the bank as unrealistic. Given this observation, we have tried to develop a rule which would reactivate the inflation target as a meaningful objective. In our setting, we do this by allowing the CB to exploit its information advantage over the shocks hitting the economy. We argue that the policy authority can bring the system closer to the desired position, if it ties its objectives to the shocks realised each time. This may not achieve the implied targets all the time but it will maximise the probability of hitting them (i.e. it will hit them on average). This is more likely to be the case when the probability of a perverse policy reaction (i.e. extreme values for a) is low. When on the other hand, this probability is substantial, Brainard's advice of greater caution in the use of the instrument is justified. Our analysis demonstrates, in an optimisation (and hence transparent) framework, why being cautious might not be the best response to uncertainty all the time. Contrary to Brainard we thus show, that when expectations are important in the determination of economic outcomes, being cautious might not be the best policy. Under certain conditions regarding the amount of uncertainty faced by the bank, the best policy would be to ignore it. There are therefore, circumstances under which, the possible costs of having to overuse the instrument are more than compensated by having private sector expectations tied to the desired target π^* effectively. The contribution of this paper is to provide an intuition which should be tested and verified in a more complex dynamic framework that allows for a greater role for expectations. It is only in such a set-up, that we would be able to judge whether the benefits of tying down private sector expectations, clearly compensate the lack of caution from the part of the central Bank and thus help manage uncertainty.

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APPENDICES

A Inflation Targeting with Parameter Certainty

Assuming certainty implies that the first two unconditional moments of the distribution of inflation can be represented by $E(\pi) = -ai$ and $var\,(\pi) = \sigma_{\varepsilon}^2$. Maximising (3) subject to (1) and (2) gives the familiar monetary policy reaction function for the interest rate and inflation:

$$i = -\frac{1}{2a}\left[\pi^* + \pi^e\right] + \frac{1}{2a}\left(2\varepsilon + \eta\right) \tag{A1}$$

$$\pi = \frac{1}{2}\left[\pi^* + \pi^e\right] - \frac{1}{2}\eta \tag{A2}$$

From (A2) expected inflation is thus equal to the target:

$$\pi^e = \pi^* \tag{A3}$$

Substituting for inflationary expectations, the Rational Expectations rules are summarised as follows:

$$i_{RE} = -\frac{1}{a}\pi^* + \frac{1}{2a}\left(2\varepsilon + \eta\right) \tag{A4}$$

$$\pi_{RE} = \pi^* - \frac{1}{2}\eta \tag{A5}$$

$$y_{RE} = \frac{1}{2}\eta \tag{A6}$$

Note that monetary policy in this context is able to counteract demand shocks in their entirety but only partially offset supply shocks. The Central Bank attains thus its first best. These are summarised in the second column of Table 1 in the main text.

A.1 The Generalised Case

As already mentioned equation (1) is a simplification of a more complex equation in which unless the interest rate moves, inflation will be equal to the level of inflation in the previous period. In other words, the generalised form of equation (1) in the main text is $\pi - \pi_{-1} = -\alpha i + \varepsilon$. In this section, we repeat the above derivations but this time for the general case. Under certainty, the first two unconditional moments of the distribution of inflation can be represented by $E(\pi) = -ai + \pi_{-1}$ and $var\,(\pi) = \sigma_{\varepsilon}^2$. Maximising (3) subject to the generalised form of (1) and (2) gives the monetary policy reaction function for the interest rate and inflation:

$$
\begin{aligned}
i &= -\frac{1}{2a}\left[(\pi^* - 2\pi_{-1}) + \pi^e\right] + \frac{1}{2a}(2\varepsilon + \eta) && \text{(A1')}\\
\pi &= \frac{1}{2}\left[(\pi^* - 2\pi_{-1}) + \pi^e\right] - \frac{1}{2}\eta + \pi_{-1} && \text{(A2')}
\end{aligned}
$$

From (A2') expected inflation is thus equal to the target:

$$\pi^e = \pi^* \qquad \text{(A3')}$$

Substituting for inflationary expectations, the Rational Expectations rules are summarised as follows:

$$
\begin{aligned}
i_{RE} &= -\frac{1}{a}(\pi^* - \pi_{-1}) + \frac{1}{2a}(2\varepsilon + \eta) && \text{(A4')}\\
\pi_{RE} &= \pi^* - \frac{1}{2}\eta && \text{(A5')}\\
y_{RE} &= \frac{1}{2}\eta && \text{(A6')}
\end{aligned}
$$

B Inflation Targeting with Brainard Uncertainty

We attempt to proceed here in a similar fashion to Brainard (1967), by introducing uncertainty in parameter a in equation (1). This type of multiplicative uncertainty is thus associated with uncertainty in the transmission process. The CB has thus only limited knowledge of the effects of its policies, as parameter a is stochastic in nature drawn from the following distribution:

$$a \rightarrow N\left(\overline{a}, \sigma_a^2\right)$$

For simplicity, we assume that a is independent of the two shocks. This time, the first two unconditional moments of the distribution of inflation are $E(\pi) = -\overline{a}i$ and $var\,(\pi) = i^2\sigma_a^2 + \sigma_\varepsilon^2$. We assume also that its coefficient of variation $\left(\frac{\sigma_a}{\overline{a}}\right)$ is sufficiently small to reduce the likelihood of having negative values for variable a. Given the stochastic nature of the policy problem, the CB formulates its policies based on the expected structure of the economy[13]. Formally it will be minimising the expected value of L.

$$E(L) = \frac{1}{2}\left\{\left[\bar{\pi} - \pi^*\right]^2 + \sigma_\pi^2 + y^2\right\}$$

[13] Our work is very similar to what Dillén and Nilsson (1998) examine, except that our optimising framework allows us to carry out a normative analysis.

For given shocks, the conditional expectation of the objective function is:

$$E(L\,|_{\varepsilon,\eta}) = \frac{1}{2}\left(\overline{\pi}-\pi^{*}\right)^{2}+\frac{1}{2}\sigma_{\pi}^{2}+\frac{1}{2}\left(\overline{\pi}-\pi^{e}+\eta\right)^{2}+\frac{1}{2}\sigma_{\pi}^{2} \tag{B1}$$

We define an increase in structural uncertainty as an increase in the variance σ_a^2. Note how there is no σ_ε^2 in the objective function, since the CB reacts to a *given* shock ε and/or η. The first two conditional moments are given now by (B2) and (B3):

$$\begin{aligned} E(\pi\,|_{\varepsilon,\eta}) &= -\overline{a}i+\varepsilon && \text{(B2)}\\ var\left(\pi\,|_{\varepsilon,\eta}\right) &= i^{2}\sigma_{a}^{2} && \text{(B3)} \end{aligned}$$

The loss function in (B1) can be rewritten as:

$$E(L\,|_{\varepsilon,\eta}) = \frac{1}{2}\left(-\overline{a}i-\pi^{*}+\varepsilon\right)^{2}+\frac{1}{2}\left(-\overline{a}i-\pi^{e}+\varepsilon+\eta\right)^{2}+i^{2}\sigma_{a}^{2} \tag{B4}$$

Optimising (B4) with respect to i gives the following policy reaction function for the instrument and inflation:

$$\begin{aligned} i &= -\frac{\overline{a}}{2\left(\overline{a}^{2}+\sigma_{a}^{2}\right)}\left[\pi^{*}+\pi^{e}\right]+\frac{\overline{a}}{2\left(\overline{a}^{2}+\sigma_{a}^{2}\right)}\left(2\varepsilon+\eta\right) && \text{(B5)}\\ E(\pi\,|_{\varepsilon,\eta}) &= \frac{\overline{a}^{2}}{2\left(\overline{a}^{2}+\sigma_{a}^{2}\right)}\left[\pi^{*}+\pi^{e}\right]+\frac{2\sigma_{a}^{2}\varepsilon-\overline{a}^{2}\eta}{2\left(\overline{a}^{2}+\sigma_{a}^{2}\right)} && \text{(B6)} \end{aligned}$$

Taking rational expectations of (B6) and simplifying we have:

$$\pi^{e} = \frac{\overline{a}^{2}}{\overline{a}^{2}+2\sigma_{a}^{2}}\pi^{*} \tag{B7}$$

Substituting (B7)[14] in (B5) and (B6) gives us the optimal equilibrium values for the interest rate, inflation and output consistent with rational expectations:

[14] For positive variation, the coefficient of $\pi^*$is less than one. This implies that the private sector expects the CB to get to something less than π^*. This is a model specific feature which has inflation at the starting point equal to zero. Even in the absence of shocks, the CB needs to undertake action in order to bring inflation to its desired position π^*.

$$i_{RE} = -\frac{\overline{a}}{\overline{a}^2 + 2\sigma_a^2}\pi^* + \frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left(2\varepsilon + \eta\right) \tag{B8}$$

$$\pi_{RE} = \frac{\overline{a}^2}{\overline{a}^2 + 2\sigma_a^2}\pi^* + \frac{2\sigma_a^2\varepsilon - \overline{a}^2\eta}{2\left(\overline{a}^2 + \sigma_a^2\right)} \tag{B9}$$

$$y_{RE} = \frac{\sigma_a^2}{\overline{a}^2 + \sigma_a^2}\varepsilon + \frac{\overline{a}^2 + 2\sigma_a^2}{2\left(\overline{a}^2 + \sigma_a^2\right)}\eta \tag{B10}$$

The results are summarised in the third column of Table 1 in the main text.

B.1 The Generalised Case

We derive again the solution for the Brainard case when equation (1) takes its generalised form. Uncertainty in parameter a implies that the first two conditional moments of the distribution of inflation are $E(\pi\mid_{\varepsilon,\eta}) = -\overline{a}i + \pi_{-1} + \varepsilon$ and $var\left(\pi\mid_{\varepsilon,\eta}\right) = i^2\sigma_a^2$. Maximising (B1) subject to the generalised form of (1) and (2), gives the monetary policy reaction function for the interest rate and inflation:

$$i = -\frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left[\left(\pi^* - 2\pi_{-1}\right) + \pi^e\right] + \frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left(2\varepsilon + \eta\right) \tag{B5'}$$

$$E(\pi\mid_{\varepsilon,\eta}) = \frac{\overline{a}^2}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left(\pi^* + \pi^e\right) + \frac{2\sigma_a^2\varepsilon - \overline{a}^2\eta}{2\left(\overline{a}^2 + \sigma_a^2\right)} + \frac{2\sigma_a^2}{2\left(\overline{a}^2 + \sigma_a^2\right)}\pi_{-1} \tag{B6'}$$

Taking rational expectations of (B6') and simplifying we get:

$$\pi^e = \frac{\overline{a}^2}{\overline{a}^2 + 2\sigma_a^2}\left(\pi^* - \pi_{-1}\right) + \pi_{-1} \tag{B7'}$$

This shows that as uncertainty increases, inflationary expectations are closer to the previous period level of inflation as the gap between that and the target does not close. Substituting (B7') in (B5') and (B6') gives us the optimal equilibrium values for the interest rate, inflation and output consistent with rational expectations:

$$i_{RE} = -\frac{\overline{a}}{\overline{a}^2 + 2\sigma_a^2}\left(\pi^* - \pi_{-1}\right) + \frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left(2\varepsilon + \eta\right) \tag{B8'}$$

$$\pi_{RE} = \frac{\overline{a}^2}{\overline{a}^2 + 2\sigma_a^2}\pi^* + \frac{2\sigma_a^2\varepsilon - \overline{a}^2\eta}{2\left(\overline{a}^2 + \sigma_a^2\right)} + \frac{2\sigma_a^2}{\overline{a}^2 + 2\sigma_a^2}\pi_{-1} \tag{B9'}$$

$$y_{RE} = \frac{\sigma_a^2}{\overline{a}^2 + \sigma_a^2}\varepsilon + \frac{\overline{a}^2 + 2\sigma_a^2}{2\left(\overline{a}^2 + \sigma_a^2\right)}\eta \tag{B10'}$$