Social Security and Growth in a Divided Society

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Abstract

We study the effects of introducing Social Security in a society with horizontal inequality. The results show an increase of capital accumulation and growth, if the beneficiary of social security subsidies cannot enter in intertemporal exchange (JEL XXX).
Introduction

A fundamental ingredient of a successful growth strategy is a significant allocation of resources away from present consumption to savings and investment. The Commission on Growth and Development call this strategy "Future Orientation" of the economy (Commission on Growth and Development 2008, p.24). In the current debate saving rates experienced by high growing Asian countries are often linked to the lack of adequate social security and social insurance institutions. In theory, the absence of social security magnifies household precautionary savings accumulation. In South Africa, the increase of saving accumulation must go hand in hand with a significant expansion of social security system as required by historical and political imperatives. In the literature it is actually not clear how social security system affect accumulation and growth, making one infers that is not social insurance per se to cause a depression of saving rates but its design and implementation.

Most saving models that account for demographics focus on how saving rates differ among people based on their age difference. In this context, demographic changes affect aggregate saving through changes in the age structure of the population, Deaton and Paxson (1997), Kelly and Schmidt (1996), Higgins and Williamson (1997) and Higgins (1998).

Beside the behavioral effect associated with the age, life expectancy is another demographic fundamental that influence saving rates and that has only been considered recently. Bloom, Canning, Mansfield and Moore (2006) analyze the impact of changes in life expectancy on life-cycle savings under different social security schemes. Based on annual data for a panel of 61 countries over the period 1960 to 2000, they conclude that the major explanation for the cross-country link between longevity and saving is the existence of retirement incentives in the social security scheme. Plus,
they find no evidence of an effect of life expectancy on saving rates in the absence of social security institutions. Along these lines, Lee, Manson and Miller (2000) use a simulation model for Taiwan and argue that the country’s saving boom over the last 40 years can be explained by an increase in longevity in a security system with fixed retirement age.

Feldstein (1977, 1995) has conducted much research on the issue of social security and savings. In his article “Social Security and Private Savings: International Evidence in an Extended Life-Cycle Model” shows that the impact of social security on private savings depends on the opposing effects of wealth replacement and induced retirement. Using data for a cross-sectional sample of developed countries he concludes that increases in social security coverage and relative levels of benefits depress the rate of private savings. Rochelle (1999) extends Feldstein’s work by accounting for the varying levels of savings across countries. Contrary to Feldstein’s results, the author concludes that the hypothesis of social security depressing savings only holds for countries with high saving rates.

Auerbach and Koflikoff (1987), Imrohoughu and Joines (1995) and Conessa and Garriga (2000) provide empirical evidence on the negative impact that introducing social security systems has on aggregate savings. However, the proposed model assumes that the households’ motive for saving is purely retirement. Such assumption can be criticized if we take into account that saving rates differ significantly with the accumulated wealth of the household as well as the introduction of a bequest motive, Barro (1974).

Fuster (2003) introduces the effects of social security taxes on the aggregate saving through their impact on labour supply. Among these lines, Nobles, Ntshongwana and Surender (2008) study the extent to which social grants in South Africa discourages workers from engaging in employment activities and instead creates a “depen-
dency culture”. The findings of the study refute the existence of such “dependency culture” among South Africans living in households that receive social grants.

In recent years, several countries have converted the financing of their social security systems from pay-as-you-go to partial or full funding. Such change is viewed as one way of “saving” social security from the political and demographic pressures that currently threaten the financial stability of unfunded systems. However, privatization would improve a nation’s situation only if such a reform increases domestic saving. Coronado (2002) uses Chile’s social security privatization to assess the impact of such a reform on household saving rates. The author found that the reform provided a significant stimulus for net of social security household saving; increasing household saving rates between 5 and 10 percentage points.

Kemnitz and Wigger (2000) study the growth and efficiency effects of pay-as-you-go financed social security when human capital is the engine of growth. They use a variant of the Lucas (1988) model with overlapping generations and show that the output growth under a properly designed unfunded social security system is higher than under a fully funded one. Plus, they found that an economy is efficient in the pay-as-you-go scheme whereas an economy with any or a fully funded system is not. The sharp contrast in their results is explained by the fact that they assumed economic growth to be driven by human capital instead of physical capital.

Bailliu and Reiser (1997) study the interaction between funded pensions and aggregate savings using panel data for eleven countries (both OECD and non-OECD) over the period 1982-1993. The authors build several proxies of pension wealth based on life insurance and internationally comparable pension fund data and prove the predictions of a simple two-period life-cycle saving model that incorporates tax treatment of pension returns, population heterogeneity, capital market imperfection and various features of pension design. They found crucial to stimulate a positive saving
impact of funded pensions from the low-saver group and to limit the negative income effect on savings by the high-saver group that emanates from the higher implicit rates of return on tax-exempt funded pensions. For that purpose, the authors propose that funded pension schemes should be mandatory rather than voluntary, that tax exemptions on pension returns should be limited to low savers and that borrowing against the accumulated mandatory pension assets should be discouraged; otherwise, funded pension schemes will fail to stimulate savings.

The Model

We first present a model without social security system.

Baseline OLG with Wealth Inequality

The analysis is based on a standard overlapping generation model augmented with horizontal heterogeneity. As the focus is on the distributive aspects of the social security reform, we consider that each age cohort is composed by two different social groups, rich and poor. Specifically, agents differ in two ways: they have different endowments of human capital, explaining the heterogeneous income distribution, and the poor don’t have access to credit. While the economy lasts into the infinite future, individuals live for two periods so that, at any point in time, the economy is composed by two generations: the young and the old. It is also assumed that the population grows at a constant rate $n$, however the stock of per capita human capital stays constant over time so as to simplify the notation of the model.

Consequently, the demographic structure of the economy is as follows. The number of individuals born in period $t$, $N_t$ will be divided between young poor and young rich. In time $(t+1)$, we will assume that the social distribution holds, so that
the young poor (rich) of period \((t)\) will be old poor (rich) in \((t + 1)\). The Government, which lasts forever and has the ability to tax and make transfers to individuals but not to consume or borrow.

**Households**

Individual households live for two periods but work only in the first one, supplying inelastically one unit of labor and earning a real wage of \(w_t\). As they retire in period \(t + 1\), they will consume part of their income in the current period and save the rest so as to finance their second-period consumption. Each individual born in period \(t\) consumes \(c_{1t}\) at time \(t\) and \(c_{2t+1}\) at time \(t+1\) and derives utility through the following life-time utility function:

\[
\begin{align*}
\nu &= \nu(c_{1t}) + \frac{1}{1+\rho}\nu(c_{2t+1}) \\
\nu(.), &> 0, \nu(.) < 0, \rho &\geq 0(1)
\end{align*}
\]

where \(\rho\) is the rate of time preference and \(\frac{1}{1+\rho}\) the subjective discount rate. Note that \(\rho\) is zero when the individuals assign the same “value” to each period, i.e when they are patient in their consumption decisions; and \(\rho\) is 1 when the individuals are impatient, assigning a higher utility to present consumption. From the moment we introduce two different groups in the analysis we need to distinguish between their utility functions and their budget constraints. Despite we could arrive to the same conclusion applying general forms, we decided to specify the utility function so as to make explicit the differences and similarities of the results with and without a social security system. As mentioned before, each generation has two representative agents, each with a particular logarithmic life –time utility function:
\[ u = \ln c_{1t} + \frac{1}{1 + \rho} \ln c_{2t+1} \]

The distinguishing characteristics of the rich and poor will be in terms of their subjective discount rate. We assume patience for the rich, $0 \leq \rho < 1$, and impatience for the poor. From the moment we are considering a heterogeneous distribution in the endowments, rich and poor will face different budget constraints.

**Rich.**—Staring by focusing in the maximization problem for the rich, the budget constraints will be as follows.

1. \[ c_{1t} = w_{1t}^R - s_t \]

2. \[ c_{2t+1} = (1 + r_{t+1})s_t \]

where $w_t$ is the real wage earned in period $t$ and $r_{t+1}$ is the interest rate paid on the saving held from period $t$ to period $t + 1$. In the second period, individuals consume all their wealth, both interest and principal. The intertemporal budget constraint is then:

\[ c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = w_{1t}^R \]

The first order conditions for this maximization problem are:

\[ \frac{dL}{dc_{1t}} = \frac{1}{c_{1t}} - \lambda = 0 \]
\[
\frac{dL}{dc_{2t+1}} = \frac{1}{(1 + \rho)(c_{1t+1})} - \frac{\lambda}{(1 + r_{t+1})} = 0
\]

Manipulating the previous equations, the FOC can be summarized to

\[(3) \quad \frac{c_{1t}}{(1 + \rho)} \frac{c_{2t+1}}{c_{2t+1}} = 1 + r_{t+1}\]

The maximum is reached when the marginal rate of substitution between present and future consumption is equal to the slope of the budget constraint.

Solving for the first period consumption and saving we arrive to equations (7) and (8):

\[
c_{1t}^R = w_{1t}^R - \frac{c_{1t}}{(1 + \rho)} = w_{1t}^R \frac{(1 + \rho)}{(2 + \rho)}
\]

\[
s_{1t}^R = w_{1t}^R - c_{1t} = w_{1t}^R \frac{1}{(2 + \rho)}
\]

From the above results we can conclude that savings increases with wage. However the effect of the interest rate is ambiguous: an increase in \( r_{t+1} \) decreases the price of second-period consumption, leading to a substitution of future for present consumption; yet, it also creates a positive income effect since the amount of saving needed to finance a given consumption in period \( (t + 1) \) reduces\(^1\).

\textit{Poor}

As previously stated, the distinguishing characteristic between rich and poor is the utility assign to present and future consumption. Assuming an impatient consumption path for those with lower endowment, the utility function will be:

\(^1\)Following Blanchard and Fischer we assume that the substitution effect dominates and an increase in the interest rate leads to an increase in saving.
\[ U = \ln c_{1t} + \frac{1}{1+\rho} \ln c_{2t+1} \]

\[ w(.) > 0, w(.) < 0, \rho = 1 \]

The poor population cannot smooth consumption over time because they do not have access to the credit market. Consumption in the second period it is given by a subsidy transfer from the government that provide a minimum consumption \( \bar{c} \).

\[ c_{1t} = w_{1t}^p \]

\[ c_{2t+1} = \bar{c} \]

\[ \text{We assume that, due to their lower wage rate and the short term sight justified in their impatient consuming behavior, young poor will not save for the second period. This issue also arrives from the existing evidence that in many developing countries, some poor people despite having demand for savings do not have access to the financial system because of their low wealth}. \]

\[ \text{The lack of saving does not mean that they will not consume, since consumption will always be positive while being alive. Notice that by being old, i.e. dead in next period and by being poor this cohort will face credit constraints and will be unable to fund the second period consumption.} \]

\[ \text{It is then, the credit market failures which exclude the poor and the paternalistic idea that the Government should help those who under-save due to forecasting errors or irrational decision, what justifies the intervention of the Government through a} \]

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\[ ^2 \text{They will maximize utility by consuming their income in period } t. \text{ However this does not mean that they wouldn’t be better off if they were able to save.} \]

\[ ^3 \text{For example banks often require minimum deposits to open accounts.} \]
social security system.

**Firm**

There are a large number of identical firms, each producing a homogenous good through a constant returns of scale Cobb-Douglas production function.

\[ Y_t = K_t^\alpha (H^R L_t^R + H^P L_t^P)^{1-\alpha} 0 < \alpha < 1 \]

\( H^i \) is the stock of human capital owned by agents of type \( i = \) rich or poor (assumed to be constant over time); \( L^i_t \) is the fraction of the population or the labor supply of type \( i \) at time \( t \) and \( K_t \) is the stock of physical capital.

Satisfying the classical assumptions for a well behaved production function, the aggregate production takes place using physical capital and two types of labor: the skilled and the unskilled, which result in different productivities. At every point in time, the firms hire workers and rent capital in competitive factor markets, and sell their output also in a competitive output market.

As they are assumed to maximize profits, the maximization problem is the following:

\[
\max K_t^\alpha (H^R L_t^R + H^P L_t^P)^{1-\alpha} - w_t^R L_t^R - w_t^P L_t^P - r_{t+1} K_t
\]

\[
\frac{dY}{dK_t} = \alpha K_t^{\alpha-1} (H^R L_t^R + H^P L_t^P)^{1-\alpha} - r_{t+1} = 0
\]

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\[ r_{t+1} = \alpha K_t^{\alpha-1}/(H^R L_t^R + H^P L_t^P)^{\alpha-1} = \alpha k_t^{\alpha-1} \]

Where \( k \) is the stock of capital per unit of effective labor.

\[ \frac{dY}{dL_t^R} = (1 - \alpha)K_t^\alpha(H^R L_t^R + H^P L_t^P)^{-\alpha}H^R - w_{1t}^R = 0 \]

\[ w_{1t}^R = (1 - \alpha)H^R K_t^\alpha/(H^R L_t^R + H^P L_t^P)^\alpha = (1 - \alpha)H^R k_t^{\alpha} \]

\[ \frac{dY}{dL_t^P} = (1 - \alpha)K_t^\alpha(H^R L_t^R + H^P L_t^P)^{-\alpha}H^P - w_{1t}^P = 0 \]

\[ w_{1t}^P = (1 - \alpha)H^P K_t^\alpha/(H^R L_t^R + H^P L_t^P)^\alpha = (1 - \alpha)H^P k_t^{\alpha} \]

Notice that if the wage rate per unit of raw labor is given by \( w_t \), then the labor income of each worker is that wage rate \( w_t \) increased by his productivity.

Going back to the equations for the intertemporal budget constraint (5), consumption (7), and saving (8) derived for the rich through the maximization problem, we can substitute \( w^R_{1t} = H^R w_t \) in.

\[ c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = w^R_{1t} = H^R w_t \]

\[ ^4 \text{Because markets are competitive, capital is paid its marginal product and since there is no depreciation, the real rate of return equals its earnings per unit of time.} \]
\[ c_{1t}^R = w_{1t}^R - \frac{c_{1t}}{1 + \rho} = H^R w_t \frac{(1 + \rho)}{(2 + \rho)} \]

\[ s_{1t}^R = w_{1t}^R - c_{1t} = H^R w_t \frac{1}{(2 + \rho)} \]

**Goods Market Equilibrium**

The good market equilibrium implies that the demand for goods in each period is equivalent to the supply. Therefore the change in the physical capital stock between period \( t \) and \( t+1 \): \( K_{t+1} - K_t \) is equal to the net aggregate saving (saving of the young and dissaving of the old).

\[ K_{t+1} - K_t = s_{it} L_t - K_t \]

Eliminating \( K_t \) from both sides, we get that the physical capital stock in \( t+1 \) is equal to the saving of the young at time \( t \):

\[ K_{t+1} = s_{it} L_t \]

However, we previously assumed that since the poor’s consumption is dominated by an impatient behavior added to their incapacity to access the financial system, their savings would be none. Moreover, only the savings that are channeled through the financial system are accumulated as physical capital and used in the production process; i.e. the capital stock is only increased through the rich’s saving.
Consequently,

\[ K_{t+1} = s_t^R L_t^R = H^R w_t \frac{1}{(2 + \rho)} L_t^R = \frac{(1 - \alpha)}{(2 + \rho)} H^R k_t^\alpha L_t^R \]

Before expressing (18) in units of effective labor, we find convenient recall the demographic structure of the economy.

Total population in period \( t \) is the summation of individual of type 1 (rich) and type 2 (poor):\(^5\)

\[ L_t = L_t^R + L_t^P \]

Of this total, a fraction \( \beta \) are the rich and \( (1 - \beta) \) are the poor:

\[ L_t = L_t^R + L_t^P = \beta L_t + (1 - \beta)L_t \]

From the initial assumptions, the population growth rate is given by \( n \) and the stock of human capital remains constant over time as well as the fraction owned by rich and poor. More precisely:

\[ L_t = (1 + n)L_{t-1} \]

\(^5\)We assume that all the population in time \( t \) supply labor in the factors market so we will use \( N_t \) and \( L_t \) indistinctively.
\[ H = H^R + H^P \]

where

\[ H^R = \sigma H \]

and

\[ H^P = (1 - \sigma)H \]

As we assume that the rich are the skilled workers, they will have a larger endowment of human capital and a higher labor income than the poor, so \( \sigma > 0.5 \).

Based on what was just mentioned, we know that

\[ H^R L^R_{t+1} + H^P L^P_{t+1} = (1 + n) \left( H^R L^R_t + H^P L^P_t \right) \]

meaning that the units of effective labor grows at rate \( n \) as well.

Expressing (18) in units of effective worker:

\[ k_{t+1} = \frac{(1 - \alpha)}{(1 + n) (2 + \rho)} \frac{H^R k_t^\alpha L^R_t}{H^R L^R_t + H^P L^P_t} \]

We know from before that:
Rearranging terms,

\[ k_{t+1} = \frac{(1 - \alpha)}{(1 + n)(2 + \rho)(1 + 2\sigma\beta - \beta - \sigma)} \sigma \beta k_t \]

Equation (25) describes the evolution of the stock of capital in terms of effective unit of labor in the scenario without a social security system. What is left to do now is find the steady state that will be then used to compare with the steady state achieved after the introduction of the social security system.

Setting \( k_{t+1} = k_t = k^* \) in equation (25) and solving for \( k^* \) we have:

\[ k^* = \left( \frac{(1 - \alpha)}{(1 + n)(2 + \rho)(1 + 2\sigma\beta - \beta - \sigma)} \sigma \beta \right)^{1/(1-\alpha)} \]

**Introducing Social Security**

We assume that government introduces a mandatory pension contribution scheme in which individuals earning income above some determined threshold are obligated to contribute a fraction \( T \) of their labour income. In this model we assume that only the young rich earn above the determined threshold and are therefore the population that are obligated to make the contribution to the pension scheme. The government
invests the contributions in individual savings account and then transfers the total contributions plus the interest payment to the young poor once they become old and retire.

**Household. — Maximization Problem for the richer cohort**

Staring by focusing in the maximization problem for the rich, the budget constraints will be as follows.

\[
(4) \quad c_{1t} = (1 - \tau) w_{1t}^R - s_t
\]

\[
(5) \quad c_{2t+1} = (1 + r_{t+1}) s_t
\]

where \(w_t\) is the real wage earned in period \(t\) and \(r_{t+1}\) is the interest rate paid on the saving held form period \(t\) to period \(t + 1\). In the second period, individuals consume all their wealth, both interest and principal. The intertemporal budget constraint is then:

\[
c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = (1 - \tau) w_{1t}^R
\]

The maximum is reached when the marginal rate of substitution between present and future consumption is equal to the slope of the budget constraint. Solving for the first period consumption and saving we arrive to the following equilibrium consumption and savings in period (1):

\[
c_{1t}^R = (1 - t) w_{1t}^R \frac{(1 + \rho)}{(2 + \rho)}
\]
\[ s_{1t}^R = (1 - t) w_{1t}^R - c_{1t}^R = (1 - t) w_{1t}^R \frac{1}{(2 + \rho)} \]

Savings of the rich cohort is obviously reduced, given the income effect. What’s happens to the aggregate savings? Note that the rich also make a contribution for their own retirement; however, as the return is equal to the return they would get in the financial market, we avoid distinguishing these from ordinary savings (we assume that they are lumped with the ordinary savings) and thus focus on the compulsory contribution.

The Consumption of the rich will reduce because of the introduction of the mandatory contribution. It will reduce by

**The Poor**

The results of the model depends critically from how the behaviour of the poor cohort changes and how this affects accumulation of capital and growth. We can analyse two hypothesis: the first, the poor are still

The poor will face the same maximization problem than before but with a new set of budget constraints:

\[ c_{1t}^p = w_{1t}^p \]

\[ c_{t+1}^p = (1 + r_{t+1}) \tau w_t^p \]

**Firm behavior**

The maximization problem of the firm is exactly as the one stated before since no changes arise due to the introduction of the social security system.
\textit{Goods Market Equilibrium}

Previously aggregating savings in the economy is required:

\[ S_t = (s_t^R + \tau w_t^R) L_t^R = \left( (1 - \tau) H^R w_t \frac{1}{(2 + \rho)} + \tau H^R w_t L_t^R \right) L_t^R \]

Notice than now a new term is added which corresponds to what the poor “indirectly save”\(^6\). The total savings of the poor equals what the government saves for them multiplied by the number of young rich that are obliged to contribute to the pension system. Similar as before, the clearing market condition is the following:

\[ K_{t+1} = \left( \frac{(1 - \tau) + (2 + \rho)\tau}{(2 + \rho)} \right) w_t H^R L_t^R = (1 - \alpha) \left( \frac{(1 - \tau) + (2 + \rho)\tau}{(2 + \rho)} \right) K_{t}^{\sigma} \sigma H L_t^R \]

Therefore we have, in per effective workers we have the aggregate capital accumulation equal to:

\[ k_{t+1} = \frac{(1 - \alpha)}{(1 + n) (1 + 2\sigma\beta - \beta - \sigma)} \left( \frac{1 + (1 + \rho)\tau}{(2 + \rho)} \right) \sigma \beta k_{t}^{\sigma} \]

Steady state

\[ k^* = \left[ \frac{(1 - \alpha)}{(1 + n) (1 + 2\sigma\beta - \beta - \sigma)} \left( \frac{1 + (1 + \rho)\tau}{(2 + \rho)} \right) \sigma \beta \right]^{1/(1 - \alpha)} \]

Comparing with the steady state without social security given by

\[ k^* = \left[ \frac{(1 - \alpha)}{(1 + n) (2 + \rho) (1 + 2\sigma\beta - \beta - \sigma)} \sigma \beta \right]^{1/(1 - \alpha)} \]

\(^6\)We use the term indirectly because it is the Government by taxing the rich the one that is saving for the poor.
we notice that the steady state shows an higher level of capital, and thus an higher level of savings during the transition path. As we can see from figure (1) with mandatory pension scheme has the extra term which will certainly positive need to be sign so as to determine the effect of the introduction of the reform on the steady state level of capital.

We know that: because \(0 < \alpha < 0\)

\(>0\) because it is defined as the fraction of the economy’s total endowment of units of effective labor owned by the rich and thus is positive. We therefore conclude that the steady state level of capital increases with the introduction of the mandatory pension contribution. Notice that as increases, the agents will earn higher incomes because wages are a positive function of the physical stock of capital. Therefore, the rich will increase their savings which will turn into to an increasing evolution of capital leading the economy to a new and higher steady state of capital.

\textit{Intertemporal Versus Intratemporal Distribution}

\textbf{Conclusions}

After our analysis, we expect that the introduction of a mandatory contribution will have the followings effects:

The poor will be better of because of the redistributive objectives of the reform added to the fact that they will receive, once retired, interest payments over “their indirect” savings. Recall that from the assumptions made, the poor didn’t have access to the financial system even if they were willing to save. Through the reform, they will have access. However notice that in our model it is not the poor who save but the rich who does the saving for them by the mandatory contribution to the Government.
Finally, they will also benefit from the changes in the payments of the labor factor. As increases, the agents will earn higher incomes because wages are a positive function of the physical stock of capital.

In term of the stock of physical capital, our model leads us to conclude that the economy will transit towards a higher steady state due to the increase in capital accumulation.

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