The Real Business Cycle Model for Lesotho

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Abstract

This paper modifies Hansen's model of indivisible labour, to include labour exports, which is parameterised and calibrated to mimic some of the Lesotho economy's distinctive features. Of particular interest in this small open economy framework, is the fact that the household's average labour supply consists of domestic and foreign components, in which the latter represents labour supplied to the South African mines. To the extent that Lesotho household's consumption expenditure, on average, exceeds domestic output, shocks to foreign labour income potentially play a critical role in influencing domestic economic activity.

The results of this neoclassical model suggest that private consumption rises while domestic output declines due to a positive shock in foreign wages, which attracts labour away from domestic production to foreign mining employment. This result has significant policy implications, in that a conventional demand management policy geared towards consumption has a potential to cause a recession. Similarly, any policy efforts to boost domestic output must be considered against the possibility of exacerbating output volatility.

1 Introduction

The socio-economic relationships between South Africa (SA) and its partners within the Common Monetary Area of Southern Africa (CMA) have potentially far reaching implications for the smaller members of the bloc: Lesotho, Namibia and Swaziland (LNS). By its nature, the CMA is an asymmetric monetary arrangement in which the LNS countries' individual currencies are pegged one to one to the SA rand. Thus SA is responsible for the monetary policy, which is managed presumably on the basis of its own domestic economic considerations. On the other hand, free capital mobility within the region imply that its partners do not have independent monetary policy. Based on the optimum currency area (OCA) theory, pioneered by Mundell (1961), the LNS countries must therefore be worse off within the CMA than they could be if they had the ability to use monetary policy countercyclically against country-specific shocks. However, the fact that the arrangement has been in existence for more than three decades[1] might be an indication that this theoretical prediction is contradicted by empirical evidence.

[1] Since the inception of the CMA, only Botswana left the arrangemnet permanently in 1976 and opted for independent monetary policy.

Table 1: Lesotho's Selected Macroeconomic Indicators: 1966 - 2007

Variable	**Ratio of GDP**
Output (GDP)	1.00
Income (GNI)	1.54
Factor Income	0.50
Consumption (private)	1.30
Investment	0.44
Imports	1.10
Exports	0.23

Thus, on the basis of the foregoing observation, this paper uses Lesotho as a case study to assess the implications of shocks from SA on Lesotho's economy.

2 Selected Economic Indicators

The fact that Lesotho's economy is closely interrelated with that of SA can be represented in different ways. For the purposes of this paper, Table 1 provides a summary of the main macroeconomic indicators: aggregate expenditures, output and national income. Based on the data from 1966 - 2007[2], the table shows the average of each variable as a ratio of GDP. From this table, one of Lesotho's distinctive economic features can be clearly identified. This relates to the fact that domestic absorption is in excess of output, with household consumption alone 1.3 times as large as GDP. This is also consistent with a large trade trade deficit, which is about 90 per cent of GDP.

One of the key sources of financing for the country's trade deficit is net factor income from abroad, which averages about 50 per cent of GDP in this sample. For many years, remittances by Basotho mine-workers in SA have been an important source of financing for domestic household expenditures. For example, see Foulo (1996) and Lucas (1987).

3 Review of Empirical Literature

4 A Model for an Economy with Labour Exports

This paper adopts Hansen's (1985) modelling framework to study a small open economy, with a special feature of labour exports. In addition to domestic labour, households provide labour services to firms located in a foreign country. For convenience, we refer to these components of labour supply simply as domestic and foreign labour, respectively. Accordingly, labour income derives from domestic as well as foreign employment . Alternatively, foreign labour supply could be considered as an export item,

[2]Although data for most variables are available from 1960, factor income is only available from 1966 while investment starts in 1970.

with consequences similar to commodity exports, at least in terms of foreign currency earnings.

4.1 Households

As is common practice in the literature, we consider an economy consisting of a continuum of identical households, indexed by $i = [0, 1]$. Labour supply - the distinctive feature of this economy - is indivisible such that an agent either works full time, for wage income, or not at all. In contrast to a model in which agents have the option of varying the number of hours devoted to labour services, in this framework, all changes in labour in the domestic economy will be indicative of fluctuations in the number of households employed at each period; that is adjustment on the extensive margin, according to Hansen (1985). However, of key interest in this paper is the foreign labour component, which is similarly assumed indivisible.

The labour indivisibility assumption is justifiable on various ground. As correctly argued by Hansen (1985), the standard practice for firms, at least in countries such as Lesotho, is to employ labour for stipulated number of hours per period. Thus job-seekers are generally not given the choice to vary the number of labour service hours but required to enter into some kind of a fixed contract. This assertion is supported by studies such as Lucas (1987) and Foulo (1996), which explain that the SA mines normally offer job-seekers from countries such as Lesotho fixed contracts per period of time, usually one calender year. Similar practice is widespread in Lesotho, where the majority of labour is employed either by the pubic sector or the textile manufacturing firms.

In the standard RBC model, with its neoclassical assumptions, implies full (non-voluntary) employment, as a result of the interaction of demand and supply forces. In contrast, the fixed contract assumption of the indivisible labour model means that at any period t some households will be employed and others unemployed. This assumption is particularly consistent with a development economy environment such as Lesotho, where the unemployment rate is officially estimated at about 23 per cent (Lesotho Bureau of Statistics, 2008).

Therefore, we employ Hansen's (1985) approach in which, instead of choosing the hours of work, households choose a probability of employment, which renders their employment prospects a subject of a random selection process. Let q_{1t} and q_{2t} be probability that a local resident will be employed at home and in a foreign country, respectively, to provide h_0^d and h_0^f fixed hours of labour services in period t. Assuming that this exhausts all the wage employment prospects for the household, the probability of non-wage employment[3] is therefore given by $1 - q_{1t} - q_{2t}$.

In each period, h_t^d hours of labour are demanded by domestic firms and h_t^f hours by foreign firms. Therefore, in each period, the equilibrium per capita hours of labour is given by:

$$h_t^d + h_t^f = q_{1t}h_0^d + q_{2t}h_0^f = h_t \tag{1}$$

[3]Non-wage employment refers to unemploment as it is conventionally known. But, in this case, it may be desirable to keep in mind that subsistence activities provide a source of livelihood for those who are not engaged in market activities.

and the household's expected labour income is:

$$E_t [w_t h_t] = w_t^d h_t^d + w_t^f h_t^f \tag{2}$$

where w_t is the wage rate at time t; superscripts d and f indicate domestic and foreign economies, respectively; and E_t is the expectations operator, conditional on information available at time t. The foreign wage rate is exogenously determined and given by:

$$\begin{aligned} \log w_t^f &= (1-\rho^w) \log \overline{w} + \rho^w \log w_{t-1}^f + \varepsilon_t^w; \\ \rho^w &\in (-1,1); \, \varepsilon_t^w \sim iidN\left(0, \sigma_w^2\right) \end{aligned} \tag{3}$$

Since the household in this model chooses consumption and the probability, and not hours, of work, its expected utility in period t is given by[4]:

$$U(c_t, q_{1t}, q_{2t}) = \log c_t + q_{1t} A_1 \log\left(1 - h_0^d\right) + q_{2t} A_2 \log\left(1 - h_0^f\right) \tag{4}$$

where $h_0^d + h_0^f = h_0$ and total time endowment is normalised to unity such that $1 - h_0$ is leisure in period t. Equation 1 implies that: $q_{1t} = \frac{h_t^d}{h_0^d}$ and $q_{2t} = \frac{h_t^f}{h_0^f}$. Therefore Equation 4 can be restated as $U(c_t, q_{1t}, q_{2t}) = \log c_t + a_1 h_t^d + a_2 h_t^f$; where $a_1 = \frac{A_1 \log\left(1-h_0^d\right)}{h_0^d}$; $a_2 = \frac{A_2 \log\left(1-h_0^f\right)}{h_0^f}$; $A > 0$; and $0 < h^j < 1$ for $j = d, f$.

In addition to labour services, households own capital stock k_t, which earns a real rental rate r_t and depreciates at the fixed annual rate δ. The stock of capital can be adjusted at a cost $\psi(k_{t+1}, k_t)$ and evolves according to the following law of motion:

$$k_{t+1} + \frac{1}{2}\psi (k_{t+1} - k_t)^2 = (1-\delta) k_t + i_t \tag{5}$$

where i_t is investment expenditure at time t; and $\psi(\bullet)$ is a function of net investment (Mendoza, 1991).

Two implications arise from the adjustment costs' functional form. First, additions and reductions to capital are equally costly and, second, these costs are an increasing function of the speed with which capital is adjusted. In small open economy models, capital adjustment costs are included to moderate investment volatility (Mendoza, 1991; Schmitt-Grohe and Uribe, 2003). In addition, capital adjustment costs make it possible to separate foreign bonds and capital in the log-linear version of the model.

It is assumed that households can buy foreign bonds b_t or borrow from the international markets at the rate r_t^f, which is given by:

$$r_t^f = r^* - \phi B_t \tag{6}$$

[4] The full period's utility function is: $U(c_t, q_{1t}, q_{2t}) = q_{1t}\left[\log c_t + A \log\left(1 - h_0^d\right)\right] + q_{2t}\left[\log c_t + A \log\left(1 - h_0^f\right)\right] + (1 - q_{1t} - q_{2t})[\log c_t + \log 1]$, which simplifies to $\log c_t + q_{1t} A \log\left(1 - h_0^d\right) + q_{2t} A \log\left(1 - h_0^f\right)$,since $\log 1$ is zero.

where r^* is the fixed world interest rate, $\phi > 0$; and B_t is the country's total stock of foreign bonds, which can be negative in cases where a country is a net debtor.

Equation 6 hypothesizes that the interest rate faced by households in the international markets increases with the country's foreign borrowing and decreases with its saving. As McCandless (2008) explains,making the rate of interest faced by domestic households a function of the country's risk premium ensures that the model has a steady state around which a log-linear approximation can be found.

The household's problem therefore is to choose a sequence of processes $\left\{b_t, c_t, h_t^d, h_t^f, k_{t+1}\right\}_{t=0}^{\infty}$ to maximise:

$$E_t \sum_{t=0}^{\infty} \left(\log c_t + a_1 h_t^d + a_2 h_t^f\right) \tag{7}$$

subject to the period-by-period budget constraint:

$$b_t + c_t + k_{t+1} + \frac{1}{2}\psi\left(k_{t+1} - k_t\right)^2 = w_t^d h_t^d + w_t^f h_t^f + r_t k_t + (1-\delta) k_t + \left(1 + r_{t-1}^f\right) b_{t-1} \tag{8}$$

and labour market clearing condition:

$$h_t^d + h_t^f = \overline{h} \tag{9}$$

where the aggregate hours of labour services are fixed[5].

A no-Ponzi game condition is also added, to rule out a possibility of ever growing consumption financed by foreign debt:

$$\lim_{t\to\infty} \frac{b_t}{\left(1 + r_t^f\right)^t} = 0 \tag{10}$$

Lett $\beta \in (0,1)$ be the household's subjective discount factor; and μ_t and λ_t represent Lagrange multipliers on the household budget constraint 8, and labour market condition 9, respectively. Then, the household optimisation problem can be formalised as:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \log c_t + a_1 h_t^d + a_2 h_t^f - \mu_t \left[\begin{array}{c} b_t + c_t + k_{t+1} + \frac{1}{2}\psi\left(k_{t+1} - k_t\right)^2 - w_t^d h_t^d \\ -w_t^f h_t^f - r_t k_t - (1-\delta) k_t - \left(1 + r_{t-1}^f\right) b_{t-1} \\ +\lambda_t \left[\overline{h} - h_t^d - h_t^f\right] \end{array} \right] \end{array} \right\} \tag{11}$$

The first order conditions associated with this problem are:

[5] The assumption of fixed aggregate hours of labour may appear too restrictive given that, in practice, agents may be required to work overtime, from time to time, for firms to meet their supply requirements. However, on the average, hours of work are assumed predetermined by either legislation, employer-labour union agreement or both.

$$\frac{\partial \mathcal{L}}{\partial c_t} : 0 = \frac{1}{c_t} - \mu_t \tag{12a}$$

$$\frac{\partial \mathcal{L}}{\partial h_t^d} : 0 = a_1 + \mu_t w_t^d - \lambda_t \tag{12b}$$

$$\frac{\partial \mathcal{L}}{\partial h_t^f} : 0 = a_2 + \mu_t w_t^f - \lambda_t \tag{12c}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} : 0 = -\mu_t \left[1 + \psi \left(k_{t+1} - k_t\right)\right] + \beta E_t \mu_{t+1} \left[\psi \left(k_{t+2} - k_{t+1}\right) + r_{t+1} + (1 - \delta)\right] \tag{12d}$$

$$\frac{\partial \mathcal{L}}{\partial b_t} : 0 = -\mu_t + \beta E_t \mu_{t+1} \left(1 + r_t^f\right) \tag{12e}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_t} : b_t + c_t + k_{t+1} + \frac{1}{2}\psi \left(k_{t+1} - k_t\right)^2 = w_t^d h_t^d + w_t^f h_t^f + r_t k_t + (1 - \delta) k_t + \left(1 + r_{t-1}^f\right) b_{t-1} \tag{12f}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : h_t^d + h_t^f = \overline{h} \tag{12g}$$

and the borrowing constraint 10.

4.2 Firms

Given $\left(w_t^d, r_t\right)_{t=0}^{\infty}$ and total factor productivity z_0 the domestic firm employs capital k_t and labour $h_{d,t}$ to produce goods and services, through a standard Cobb-Douglas production technology:

$$y_t = z_t k_t^{\theta} h_{d,t}^{1-\theta} \tag{13}$$

where $\theta \in (0, 1)$;and the log of factor productivity follows a first order autoregressive AR(1) process:

$$\begin{aligned} \log z_t &= (1 - \rho^z) \log \overline{z} + \rho^z \log z_{t-1} + \varepsilon_t^z \\ \rho^z &\in (-1, 1) ; \varepsilon_t^z \sim iidN\left(0, \sigma_z^2\right) \end{aligned} \tag{14}$$

while capital accumulation is governed by the process described in equation 5.

Therefore, the representative firm's problem is to solve:

$$\max_{h_t^d, k_t} \Pi = z_t k_t^{\theta} h_{d,t}^{1-\theta} - w_t^d h_{d,t} - r_t k_t \tag{15}$$

The resultant first order conditions are:

$$\theta z_t k_t^{\theta-1} h_{d,t}^{1-\theta} = r_t \tag{16a}$$

$$(1 - \theta) z_t k_t^{\theta} h_{d,t}^{-\theta} = w_t^d \tag{16b}$$

These are the standard firm's demand curves, under the neo-classical assumption of perfect competition, indicating that factors of production are paid their marginal physical products.

4.3 Market Clearing Conditions

Let capital letters represent aggregate variables. Then, with a continuum of households defined on a unit interval, in equilibrium, all the aggregate variables will behave like their individual counterparts. That is,in aggregate terms,

$$
\begin{aligned}
B_t &= b_t \\
C_t &= c_t \\
H_t &= h_t \\
K_t &= k_t \\
I_t &= i_t
\end{aligned}
$$

The economy's resource constraint is found by aggregating the household budget constraint 8 as follows:

$$C_t + I_t + B_t = w_t^d H_t^d + r_t K_t + w_t^f H_t^f + \left(1 + r_{t-1}^f\right) B_{t-1} \tag{17a}$$

or

$$C_t + I_t + (B_t - B_{t-1}) = Y_t + w_t^f H_t^f + r_{t-1}^f B_{t-1} \tag{17b}$$

where $I_t = K_{t+1} + \frac{1}{2}\psi\left(K_{t+1} - K_t\right)^2 - (1-\delta) K_t$ is aggregate investment expenditure. Equation 17b follows from the the zero profit assumption of a linearly homogeneous production technology. It indicates that, for an economy without government expenditure, its gross national income (GNI) - on the right hand side - can be spend on consumption, investment or acquisition of foreign claims. As explained in the following paragraphs, these net acquisition of foreign claims are a result of net exports of goods and services. The GNI comprises gross domestic product (GDP), represented by Y_t, and factor income from abroad due to labour $w_t^f H_t^f$ and capital $r_{t-1}^f B_{t-1}$ services.

In the context of an open economy, a balance of payments (BOP) condition is also required to clear the foreign exchange market. Since it is embedded in equation 17b, it is instructive to derive the BOP condition directly from the total resource constraint. That is:

$$(B_t - B_{t-1}) = Y_t + w_t^f H_t^f + r_{t-1}^f B_{t-1} - C_t - I_t \tag{18}$$

This equation shows that, on the left hand side, changes in a country's net foreign asset position are equal to the difference between its aggregate income $Y_t + w_t^f H_t^f + r_{t-1}^f B_{t-1}$ and expenditure $C_t + I_t$. Using the national income accounting identity $Y_t = C_t + I_t + X_t$, we can substitute net exports of goods and services X_t for $Y_t - C_t - I_t$ in equation 18 to get the required BOP condition:

$$(B_t - B_{t-1}) = X_t + w_t^f H_t^f + r_{t-1}^f B_{t-1} \tag{19}$$

which is the standard constraint that an open economy can have a current account surplus or deficit, on the right hand side of 19 that would be reflected by a change in net foreign assets. A current account surplus means that the domestic economy is acquiring claims on foreigners while a deficit is an indication of borrowing from the international markets. Finally, the aggregate labour market clearing condition is given by:

$$H_t^d + H_t^f = \overline{H} \tag{20}$$

4.4 The Model in Aggregate Terms

We use equation 6 to eliminate r^f and the first order conditions 12a - 12c to eliminate the Lagrange multipliers from the system. The full model comprises 9 variables: $C_t, B_t, K_{t+1}, H_t^d, H_t^f, r_t, w_t^d, X_t, Y_t$ and 2 stochastic processes: w_t^f and z_t. That is,

$$\alpha_1 + \frac{w_t^d}{C_t} = a_2 + \frac{w_t^f}{C_t} \tag{21a}$$

$$\frac{1}{\beta}\left[1 + \psi\left(K_{t+1} - K_t\right)\right] = E_t \frac{C_t}{C_{t+1}}\left[\psi\left(k_{t+2} - k_{t+1}\right) + r_{t+1} + (1-\delta)\right] \tag{21b}$$

$$E_t \frac{C_{t+1}}{C_t} = \beta\left(1 + r^* - \phi B_t\right) \tag{21c}$$

$$r_t = \theta z_t K_t^{\theta-1} H_{d,t}^{1-\theta} \tag{21d}$$

$$w_t^d = (1-\theta)\, z_t K_t^{\theta} H_{d,t}^{-\theta} \tag{21e}$$

$$B_t + C_t + K_{t+1} + \frac{1}{2}\psi\left(K_{t+1} - K_t\right)^2 = Y_t + w_t^f H_t^f + (1-\delta)\, K_t + \left(1 + r^* - \phi B_{t-1}\right) B_{t-1} \tag{21f}$$

$$B_t - B_{t-1} = X_t + w_t^f H_t^f + \left(r^* - \phi B_{t-1}\right) B_{t-1} \tag{21g}$$

$$Y_t = z_t K_t^{\theta} H_{d,t}^{1-\theta} \tag{21h}$$

$$H_t^d + H_t^f = \overline{H} \tag{21i}$$

and

$$\lim_{t\to\infty} \frac{B_t}{\left(1 + r_t^f\right)^t} = 0 \tag{21j}$$

Most of the first order conditions are standard in open economy RBC models. However, a few of them deserve special attention. Equation 21a equates the marginal benefit of domestic work effort to that of foreign labour. Noting that α_1 and α_2 represent the respective marginal disutilities of labour, this condition shows that the optimal domestic and foreign wage rates would be equal if α_1 and α_2 were the same. But, based on the assumption that foreign labour supply is associated with considerably higher disutility than domestic work effort[6], equation 21a indicates that the foreign wage rate must be higher than its domestic counterpart for this condition to hold. Intuitively, a rational agent would only be interested in seeking employment in

[6] For instance, as Lucas (1987) explains, costs associated with mine labour include (a) miners returning with diseases such as TB and (b) the proliferation of single-parent households.

a foreign country if the expected wage is high enough to compensate for the relatively higher costs associated with foreign employment[7].

As indicated equations 21f and 21g highlight, households' income in a labour-exporting country includes foreign wages. Therefore, it is feasible in this economy to simultaneously run a trade deficit ($X < 0$) and accumulate foreign reserves ($B_t - B_{t-1} > 0$), depending on the relative sizes of labour income ($\overline{W}^f \overline{H}^f$) and any payments to foreign entities. As demonstrated by the data in Table 1, net factor income from abroad represents a significant share of Lesotho's economic activity, which is an issue to which we return later. This is in contrast with the conventional way of thinking about the balance of payments constraint, where a country's current account balance is dominated by its trade account, with deficits financed by borrowing from the rest of the world. Finally, equation 21i is just the aggregate form of the household's labour market clearing condition 1, which divides labour into domestic and foreign components.

4.5 Loglinear Approximation

4.5.1 Stationary State

The steady state solution of the model, $\overline{X} = X_t = X_{t+j}$ for any variable X_t and all $j \in \mathbb{Z}$, is as follows:

$$\alpha_1 + \frac{\overline{w}^d}{\overline{C}} = a_2 + \frac{\overline{w}^f}{\overline{C}} \tag{22a}$$

$$\frac{1}{\beta} = \overline{r} + (1 - \delta) \tag{22b}$$

$$\frac{1}{\beta} = 1 + r^* - \phi \overline{B} \tag{22c}$$

$$\overline{r} = \theta \overline{z} \left(\frac{\overline{H}_d}{\overline{K}} \right)^{1-\theta} \tag{22d}$$

$$\overline{w}^d = (1 - \theta)\, \overline{z} \left(\frac{\overline{K}}{\overline{H}_d} \right)^{\theta} \tag{22e}$$

$$\overline{C} + \delta \overline{K} = \overline{Y} + \overline{w}^f \overline{H}^f + \left(r^* - \phi \overline{B} \right) \overline{B} \tag{22f}$$

$$0 = \overline{X} + \overline{w}^f \overline{H}^f + \left(r^* - \phi \overline{B} \right) \overline{B} \tag{22g}$$

$$\overline{Y} = \overline{z} \overline{K}^{\theta} \overline{H}_d^{1-\theta} \tag{22h}$$

$$\overline{H} = \overline{H}^d + \overline{H}^f \tag{22i}$$

The steady state solution 22a-22i has enough information to solve for the stationary values of our model. From equations 22b and 22c it is straight forward to determine $\overline{r}$ and $\overline{B}$ as:

[7] Of course it is implicitly assumed that one has a choice and thereby ruling out existence of excess labour supply, which would be the most likely situation in a developing country scenario.

$$
\begin{aligned}
\overline{r} &= \frac{1}{\beta} - (1-\delta) && (23)\\
\overline{B} &= \frac{\left(1 + r^* - \frac{1}{\beta}\right)}{\phi} && (24)
\end{aligned}
$$

Then 22g will determine $\overline{X}$, given $\overline{w}^f$ and $\overline{H}^f$.

The factor market conditions 22d and 22e yield the stationary state domestic wage rate as:

$$\overline{w}^d = (1-\theta)\left(\frac{\theta}{\overline{r}}\right)^{\frac{\theta}{1-\theta}} \quad (25)$$

Once, the domestic wage rate and labour are determined, capital can be solved as:

$$\overline{K} = \left(\frac{\overline{w}^d}{1-\theta}\right)^{\frac{1}{\theta}} \overline{H}^d \quad (26)$$

In line with labour indivisibility assumption, aggregate hours of work account for one third of total time endowment:

$$\overline{H}^d + \overline{H}^f = \overline{H} = 0.333 \quad (27)$$

and $\overline{H}^d$ and $\overline{H}^f$ can be determined from the empirical data to resemble long run average shares of domestic and mine employment, respectively. Then, the rest of the stationary values are directly obtainable from the relevant equations. In particular, equation 22a determines the value of steady state consumption as a function of the differential between the foreign and the domestic wage rate:

$$\overline{C} = \frac{\overline{w}^f - \overline{w}^d}{\alpha_1 - a_2} \quad (28)$$

4.5.2 Log-linearisation

Following Uhlig (1999), let $\widetilde{x}_t = \log\left(X_t/\overline{X}\right)$, where $\overline{X}$ stands for steady state value of a variable X_t. Then, the equilibrium conditions of our model can be solved in log-deviation from their steady states: $\widetilde{c}_t, \widetilde{k}_{t+1}, \widetilde{b}_t, \widetilde{r}_t, \widetilde{h}_t^d, \widetilde{h}_t^f, \widetilde{w}_t^d, \widetilde{x}_t$ and $\widetilde{y}_t$ as follows:

$$\widetilde{c}_t = \frac{\overline{w}^f \widetilde{w}_t^f - \overline{w}^d \widetilde{w}_t^d}{\overline{w}^f - \overline{w}^d} \tag{29a}$$

$$\overline{K}\left(1+\beta\right)\psi\widetilde{k}_{t+1} = \widetilde{c}_t - E_t\widetilde{c}_{t+1} + \overline{K}\psi\widetilde{k}_t + \beta\psi\overline{K}E_t\widetilde{k}_{t+2} + \beta\overline{r}E_t\widetilde{r}_{t+1} \tag{29b}$$

$$\beta\phi\overline{B}\widetilde{b}_t = \widetilde{c}_t - E_t\widetilde{c}_{t+1} \tag{29c}$$

$$\widetilde{r}_t = \widetilde{z}_t + \left(\theta-1\right)\widetilde{k}_t + \left(1-\theta\right)\widetilde{h}_t^d \tag{29d}$$

$$\theta\widetilde{h}_t^d = \widetilde{z}_t + \theta\widetilde{k}_t - \widetilde{w}_t^d \tag{29e}$$

$$\overline{B}\widetilde{b}_t + \overline{C}\widetilde{c}_t + \overline{K}\left[\widetilde{k}_{t+1} - \left(1-\delta\right)\widetilde{k}_t\right] = \overline{Y}\widetilde{y}_t + \overline{w}^f\overline{H}^f\left(\widetilde{w}_t^f + \widetilde{h}_t^f\right) + \left[\left(1+r^*\right)\overline{B} - 2\phi\overline{B}^2\right]\widetilde{b}_{t-1} \tag{29f}$$

$$\overline{B}\widetilde{b}_t = \overline{X}\widetilde{x}_t + \overline{w}^f\overline{H}^f\left(\widetilde{w}_t^f + \widetilde{h}_t^f\right) + \left[\left(1+r^*\right)\overline{B} - 2\phi\overline{B}^2\right]\widetilde{b}_{t-1} \tag{29g}$$

$$\widetilde{y}_t = \widetilde{z}_t + \theta\widetilde{k}_t + \left(1-\theta\right)\widetilde{h}_t^d \tag{29h}$$

$$\overline{H}^d\widetilde{h}_t^d = -\overline{H}^f\widetilde{h}_t^f \tag{29i}$$

The two stochastic variables follow a first-order autoregressive process:

$$\widetilde{w}_t^f = \rho^w\widetilde{w}_{t-1}^f + \varepsilon_t^w \tag{30a}$$

$$\widetilde{z}_t = \rho^z\widetilde{z}_{t-1} + \varepsilon_t^z \tag{30b}$$

where the distributions of ε_t^w and ε_t^z are given by 3 and 14, respectively.

In passing, we note that it might be more convenient to restate equation 29a, using 28, as:

$$\left(\alpha_1 - a_2\right)\overline{C}\widetilde{c}_t = \overline{w}^f\widetilde{w}_t^f - \overline{w}^d\widetilde{w}_t^d$$

4.6 Calibration of the Model

In order to solve the model, values must be chosen for different parameters. Tables 2 and 3 contain, respectively, parameter and steady state values used in calibration of the model. The basic parameters for RBC models are well established, particularly, in the case of industrial countries. Mendoza (1995) also has sub-groups of developing countries including some from Africa, which may be relevant to this study. However, the core parametarisation strategy of this paper is to make the steady state solution of the model roughly similar to the long-run average values of Lesotho's key macroeconomic indicators, as elaborated in the rest of this section.

Our theoretical model does not have government,and we thus exclude government expenditure from Lesotho's GDP to align the model's steady state values with their empirical counterparts. As indicated in Table 1, income to Lesotho residents deriving from mine labour in SA is economically very significant. Due to the pivotal role of foreign labour income in this paper, the model's parameters are selected to be consistent with Lesotho's average net factor income for the period 1998 - 2007, which is 34 per cent of GDP, after excluding government. One set of parameters and steady states is predetermined, on the basis of empirical data and the literature. The other set is then solved endogenously, as a function of the first one.

The reference period is chosen to reflect the most recent economic situation in Lesotho, as some of the economy's key structural features have changed significantly overtime. For instance, based on the unadjusted Lesotho data, the mean of labour income to GDP is about 50 per cent for 1966 - 2007, 37 per cent for 1988 - 2007 and only about 25 percent for the latest 10-year period. The latter corresponds to 34 per cent of GDP when government expenditure is excluded.

The following are selected from the empirical data. The average real rate of interest, $\overline{r}$, in Lesotho for the 10 years to 2007 is 8.5 per cent. Its US counterpart, which serves as a proxy for world interest rate, r^*, is about four per cent. Both are obtained from the World Bank's development indicators. $\overline{H}^f$ and $\overline{H}^d$ are based on the most recent official labour statistics in Lesotho. Data for Basotho mine-workers in SA is obtainable from the Central Bank of Lesotho's (CBL's) publications. Total employment for the whole country is based on the Lesotho Bureau of Statistics (BOS) 2008 labour force survey. From these sources, we estimate the share of mine-workers to total number of Basotho employed, $\overline{H}^f$, to be about 10 per cent; therefore the domestically employed labour, $\overline{H}^d$, account for about 90 per cent, on the average.

The value for the share of capital to income, θ, is 0.4. Based on the literature, this value seems to correspond closely to the advanced countries. Given high rates of unemployment in the developing countries, labour's share of aggregate income should presumably be much smaller than in the high income countries. For example, Mendoza's (1995) labour share for the developing countries averages about 36 per cent. However, for our purpose, the steady state share of capital is determined such that the domestic wage rate $\overline{w}^d$ is about one fifth of the foreign wage rate $\overline{w}^f$, given $\overline{r}$ (see equation 25). Although data availability is a problem, anecdotal evidence suggests that the nominal wage in SA mines should be much higher than the alternative domestic wage.

The household's subjective discount factor $\beta = 0.97$ is chosen such that, given $\overline{r}$, the annual rate of depreciation δ would be around 5 per percent. A few studies such as Hansen (1985) and Mendoza (1991) set the depreciation rate of 10 per cent. Assuming the existence of a positive relationship between capacity utilisation and the rate at which capital stock is run down, it is not implausible to hypothesise a lower rate of depreciation in the context of a developing economy than in the industrial countries. Even, in the case of high income countries, King and Rebelo (2000) indicate that there is evidence to suggest that the depreciation rate is lower than previously estimated.

We set ϕ close to zero, in line with Uribe (2002), to ensure that the model behaves as if the foreign rate of interest is exogenous. The capital adjustment cost parameter is set $\psi = 0.028$, following Mendoza (1991). The standard deviations of output σ_z and foreign wage σ_w are estimated from detrended output and miners' earnings, respectively. In contrast, calibration of the persistence parameters ρ^z and ρ^w is informed by the need to match the model's response to shocks to key Lesotho's empirical regularities. Therefore, we set $\rho^z = 0.9$ and $\rho^w = 0.8$.

Given the predetermined subset of parameters, as explained above, the rest follow logically from the steady state solutions in section **??**. The net foreign asset position $\overline{B}$ is determined by equation 24 to be positive. This is consistent with the fact that the

Table 2: Parameters

Parameter	Value	Description
α_1	-2.0	disutility of domestic labour
α_2	-8.2	disutility of mine labour
β	0.97	subjective discount factor
δ	0.054	annual rate of depreciation
θ	0.40	capital's share in output
ϕ	0.01	debt elasticity of interest rate premium
ψ	0.028	capital adjustment cost parameter
r^*	0.04	fixed world rate on interest
ρ^w	0.90	AR(1) coefficient of foreign wage rate
ρ^z	0.80	AR(1) coefficient of total factor productivity
σ_w	0.042	standard deviation of foreign wage rate
σ_z	0.043	standard deviation of total factor productivity

Table 3: Steady State Values

$\overline{B}$	$\overline{C}$	$\overline{FI}$	$\overline{K}$	$\overline{H}^d$	$\overline{H}^f$	$\overline{H}$	$\overline{X}$	$\overline{Y}$	$\overline{w}^d$	$\overline{w}^f$	$\overline{r}$
0.91	0.91	0.28	3.94	0.298	0.035	0.333	-0.28	0.84	1.68	7.33	0.085

steady state net rate of return on capital, $\overline{r} - \delta = 0.0309$, is less than the world rate of interest, $r^* = 0.04$. Therefore, in equilibrium, residents will save in foreign markets to equalise the domestic and foreign rates of return.

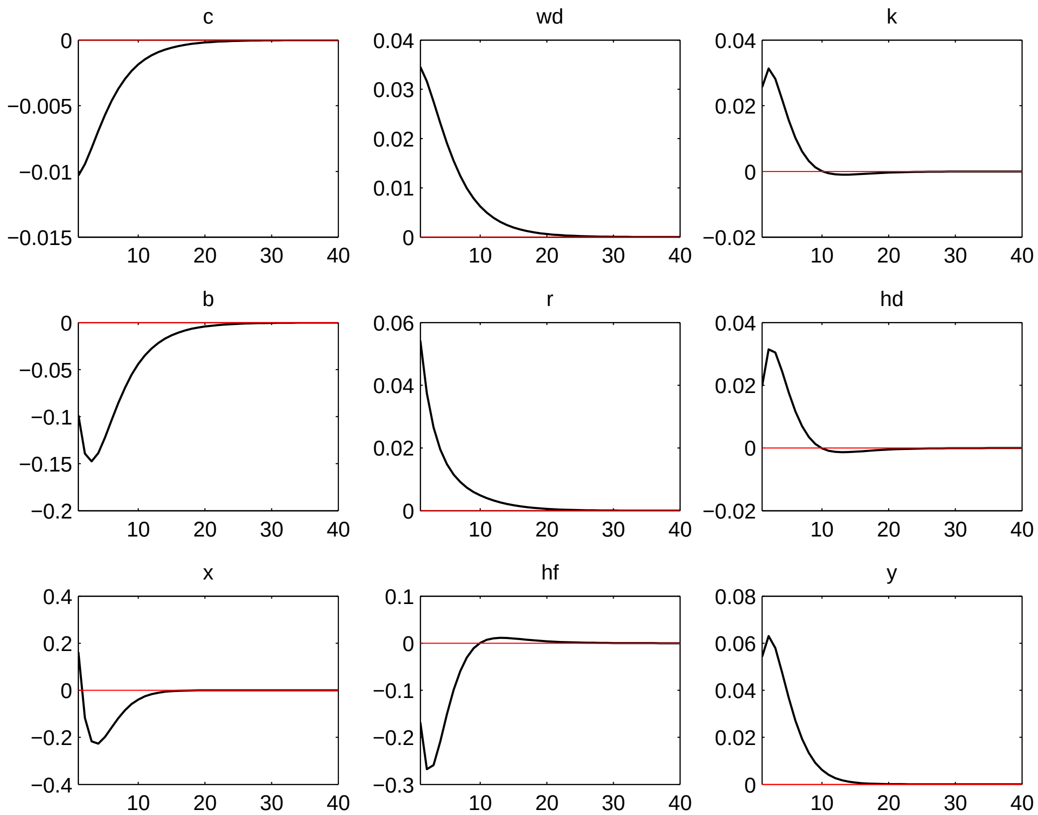
The domestic wage rate, $\overline{w}^d = 1.68$, is given by equation 25. In turn, the equilibrium capital stock $\overline{K} = 3.94$ is determined from equation 26. This implies that the model's steady state investment is about 25 per cent of GDP, which is roughly close to the most recent 10-year average private investment in Lesotho. Solution of the $\overline{K}$ provide enough information to determine the steady state aggregate output, $\overline{Y} = 0.84$, from the production function, where total factor productivity $\overline{z}$ is set equal to unity. As already indicated, the model is calibrated such that net factor income from abroad is 34 per cent of GDP. This implies that $\overline{w}^f \overline{H}^f + \left(r^* - \phi \overline{B}\right) \overline{B} = 0.28$ and thus net exports $\overline{X} = -0.28$ (equation 22g). The net factor income equation can now be solved for $\overline{w}^f$, which is found to be about 7.33. This translates into a ratio of about 4.3 between miners' wage and domestic wage rates.

Consumption is solved as a residual from the aggregate resource constraint (equation 22f) and given as $\overline{C} = 0.91$. This translates into about 1.1 as a ratio of GDP, which is somewhat smaller than the empirical average of about 1.5. However, fixing the model's steady state factor income to match the data means that at least one of the variables should be a residual. The fact that the model's equilibrium consumption is still 10 per cent above GDP is taken to be sufficient to reasonably approximate Lesotho's economy. Finally, $\alpha_1 - \alpha_2$ is given by equation 28. We set $\alpha_1 = -2$, which is in line with Hansen (1985). This, in turn, solve for α_2 as -8.2, implying that the disutility of mine labour is about 4 times larger than that of domestic labour, which is consistent with the ratio the corresponding wage rates.

5 Results

5.1 Impulse Responses to Total Factor Productivity (TFP) Shocks

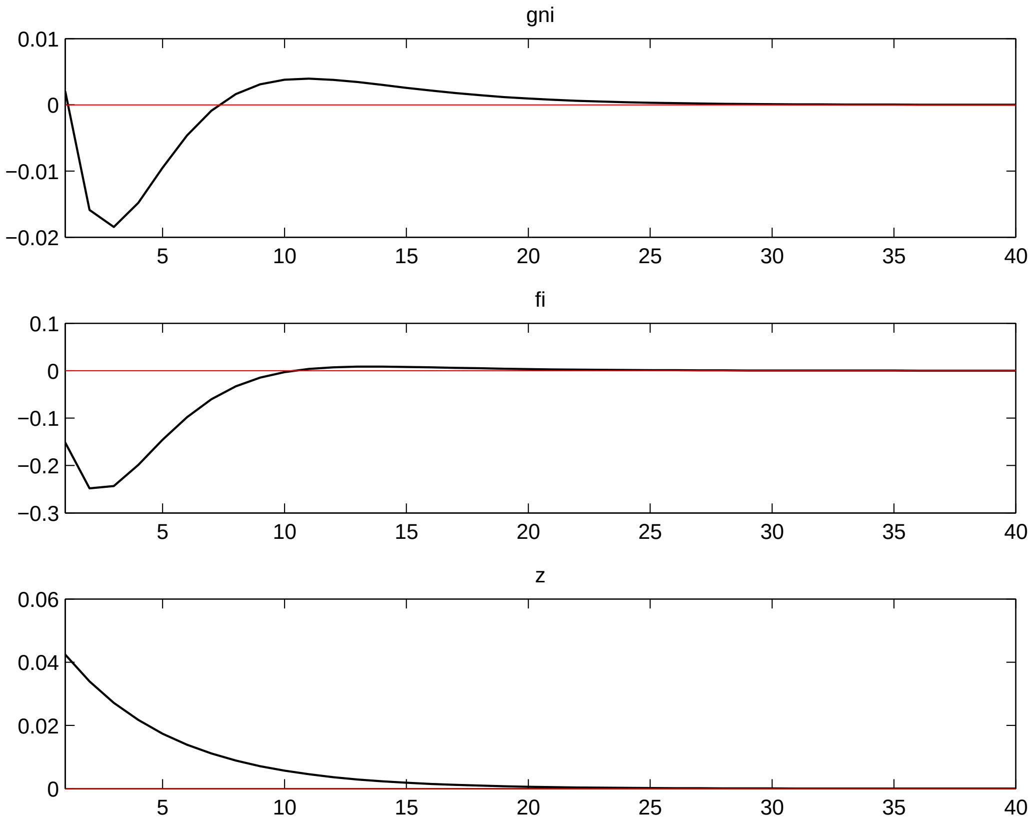
Figure 1: Impulse Responses to Factor Productivity Shocks



The basic RBC model used in this paper cannot be expected to mimic all empirical features of the Lesotho economy. In particular, the model's simplifying assumption of perfect labour mobility between the domestic and foreign markets induces excessive volatility in response to stochastic shocks. However, the qualitative results of this model have a potential to change the way in which the economy of Lesotho should be analysed, mainly, due to its strong reliance on SA. The remainder of this section discusses the qualitative response of the model economy to exogenous shocks.

As a starting point, in line with the standard practice in the RBC literature, we analyse the impact of total factor productivity (TFP) shocks on the real economy, before turning attention to the implications of foreign wage shocks. Figures 1 and 2 show the impulse responses of the model's variables to a one standard deviation shock to TFP. In Figure 1, a positive shock to TFP increases domestic output $\widetilde{y}_t$ and labour $\widetilde{h}_t^d$, and capital $\widetilde{k}_t$. Domestic factor prices also rise, as GDP increases faster than labour and capital and thereby raising factor productivity. The response of output, and factor prices, is consistent with theoretical expectations.

Figure 2: Impulse Responses to Factor Productivity Shocks



A rise in domestic labour, in turn, implies a reduction in foreign labour supply $\widetilde{h}_t^f$, due to the assumption of fixed total hours of labour. As in reflected in Figure 2, this leads to a sharp decline in net factor income from abroad, which ultimately drags GNI down. However, the initial impact of productivity shock leaves GNI almost unchanged, which implies that the responses of output and net factor factor income initially balance each other out. Subsequently, as GDP returns to its steady state level, GNI begins to fall and reaches a trough within two and half years.

One interesting aspect relates to the seeming differences in the speed of adjustment of GDP, on the one hand, and factor income and GNI, on the other. While factor income returns to its long run equilibrium level by the tenth period, GDP takes a few more periods and returns to its steady state only around the fifteenth year. As a result, GNI moves marginally above its long run level by the eighth year and seems to remain in that position for a considerably long period.

On the demand side, the model's prediction is that a positive productivity shock will result in a decline in private consumption $\widetilde{c}_t$ and net foreign assets $\widetilde{b}_t$ but an improvement in net exports $\widetilde{x}_t$. This pattern of responses by the private agents in the model is indicative of the distinctive feature of the economy of Lesotho and can better be evaluated against the dependency of this economy on SA, as outlined in section 2. As explained in that section, the long run aggregate consumption exceeds GDP, and the resultant trade deficit is financed in part by mine-workers' remittances. Therefore, the decline in consumption following a rise in domestic output, driven by a productivity shock, is consistent with the model's assumption that the foreign wage rate is much higher than the domestic wage rate, to compensate for the relatively higher disutitility of foreign labour. The effect of a rise in domestic labour productivity, therefore, is to narrow the foreign-domestic wage differential. To some extent, this reduces the attractiveness of foreign labour in favour of the domestic economy. However, the associated increase in domestic factor income is not enough to offset the decline in foreign labour income, which translates into less aggregate income for the economy.

As already indicated, the response of the model's external sector is also not entirely consistent with the conventional theoretical expectation. In figure 1, a positive TFP shock induces a reduction in net foreign assets $\widetilde{b}_t$ while net exports $\widetilde{x}_t$ initially improve. A fall in consumption in an import-dependent economy can be expected to result in a reduction in consumption-related imports. Therefore, the temporary improvement in the trade balance means that a TFP shock results in a larger decline in consumption goods than any increase in input factors.

The impulse response of the economy's net foreign asset position can best be explained using equation 29g, suitably rearranged as:

$$\overline{B}\left(\widetilde{b}_t - \widetilde{b}_{t-1}\right) = \overline{X}\widetilde{x}_t + \left[\overline{w}^f \overline{H}^f \left(\widetilde{w}_t^f + \widetilde{h}_t^f\right) + \left(r^* \overline{B} - 2\phi \overline{B}^2\right) \widetilde{b}_{t-1}\right]$$

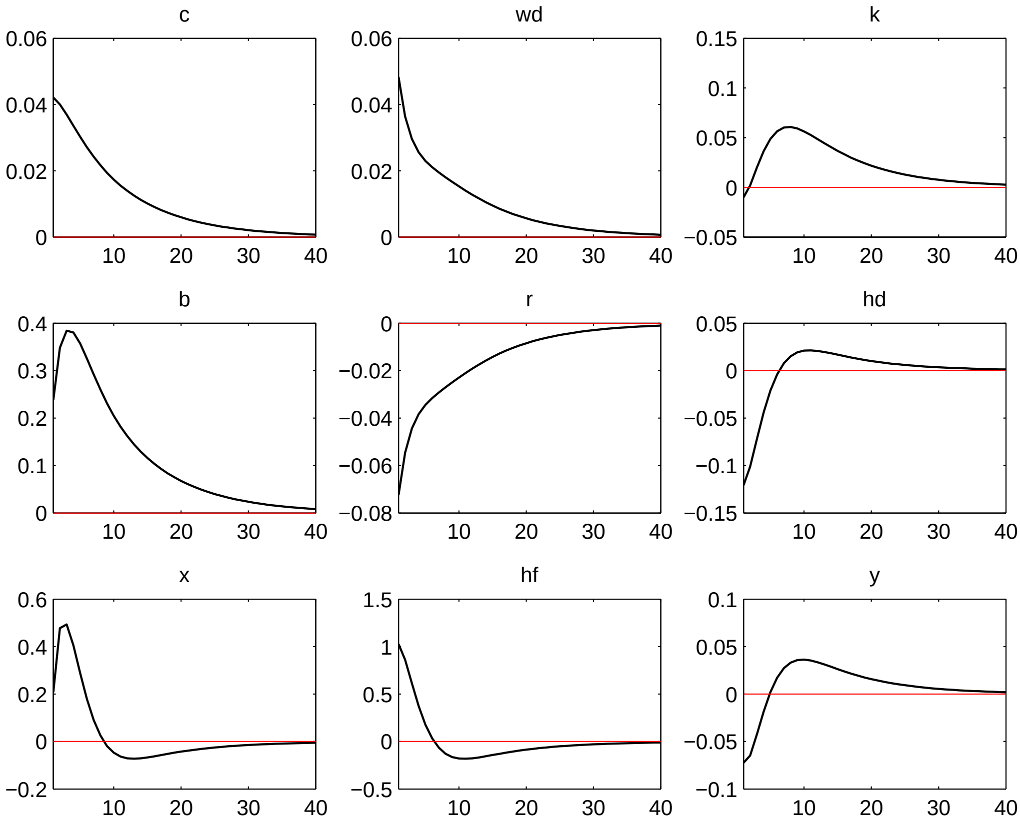
where the term in square brackets on the right hand side represents net factor income from abroad. Based on this expression, a draw-down on foreign exchange reserves, $\widetilde{b}_t - \widetilde{b}_{t-1} < 0$, that is accompanied by a combination of rising net exports and falling factor income means that the improvement in the trade balance is relatively smaller than the reduction in foreign factor income. A plausible economic interpretation seems to suggest that, as consumption falls, due to a decrease in net factor income, imports also decline to some extent. However, since domestic production increases, the economy still requires to import some of the productive inputs and thereby counterbalances the reduction of consumption imports. In the face of falling foreign labour income, imports will be financed by capital inflows or a reduction in the country's foreign exchange reserves.

An alternative explanation is that households seem to smooth their consumption, in the face of falling labour income, by reducing their foreign savings. This interpretation is consistent with the fact that the reduction in consumption is relatively much smaller than that in factor income.

5.2 Impulse Responses to Foreign Wage Shocks

The responses of our model variables to a shock to the foreign wage rate $\widetilde{w}_t^f$ are summarised by figures 3 and 4. As expected, a positive shock to the foreign wage rate raises net factor income from abroad and GNI (figure 4). However, as figure 3 shows, output $\widetilde{y}_t$ declines due to a switch from domestic to foreign labour supply. It nonetheless returns to and overshoots its steady state level by around the fifth period. Thus the increase in GNI suggests that the initial growth in factor income outweighs the decline in GDP. In passing, we note that the negative response by GDP to a positive foreign-wage shock may simply be driven by the model's unduly restrictive

Figure 3: Impulse Responses to Foreign Wage Shocks

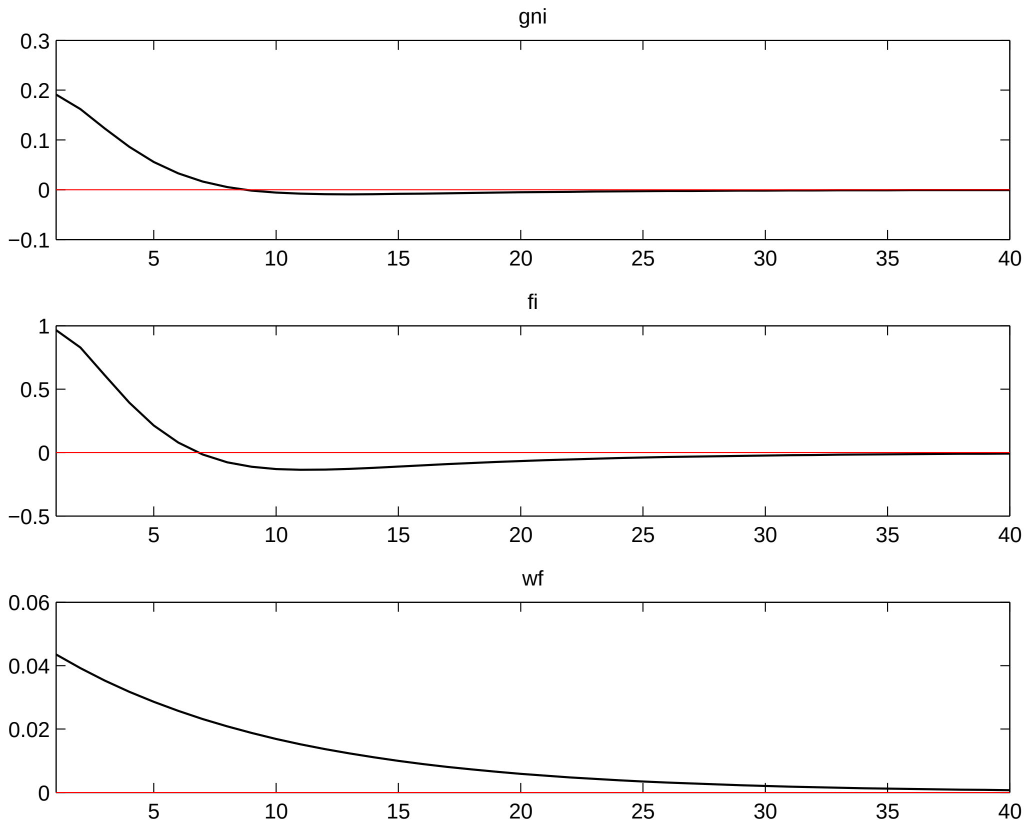


labour market condition. In this regard, the domestic wage rate rises with labour productivity that is spurred by a sharper decline in domestic labour than GDP.

The first column of figure 3 shows the impulse responses by consumption $\widetilde{c}_t$, foreign bond holding $\widetilde{b}_t$ and net exports $\widetilde{x}_t$. To a large extent, the responses of these three variables underline the strong influence of foreign labour income on our model economy. It is noteworthy to note that all three respond positively to a foreign wage shock. Firstly, in contrast with the initial negative effect on GDP, consumption increases significantly, due to the strong positive effect on net factor income and thus GNI. One outstanding feature is that consumption seems to remain above its long run level for a long time (more than 40 periods), after the initial impulse. This is in contrast with the relatively temporary increases in factor income and GNI, which last for less than 10 years. The tendency by the model's consumption to move relatively slowly towards its long run level, after a foreign wage shock, may explain the relatively quick recovery in GDP.

Secondly, the model suggests that the economy saves part of the windfall from foreign labour income by increasing its foreign bond-holding. In addition, despite a rise in consumption, net exports also increase. It is instructive to interpret this result against that in section 5.1. As shown in figure 1, consumption and bonds respond negatively to a TFP shock, while the trade balance initially improves. In contrast, all three respond positively to the foreign wage shock (figure 3). The rise in foreign bonds

Figure 4: Impulse Responses to Foreign Wage Shocks



is in line with theoretical expectations and suggests that agents respond to an unanticipated increase in income by partly raising their consumption and partly increasing their saving. This is also consistent with a consumption smoothing behaviour, where part of the windfall is used to increase foreign reserves, which are then used to cushion the blow on consumption during the economic downturn.

The improvement in the trade balance results from the fact that the initial reduction in output seems to be significantly larger than the increase in consumption. Since the model economy envisaged in this paper is heavily dependent on imports, in line with Lesotho's economy, imports of raw materials and other intermediate goods may fall with output. On the other hand, given the posited large import content in the economy's consumption basket, increasing consumption should pull up imports. In this case, the significant positive response by net exports suggests a stronger procyclical correlation of imports and output than consumption. Interestingly, although both TFP and foreign wage shocks initially induce positive trade balance response, the effect of the wage shock is much stronger and long-lasting. This is in contrast with a relatively short-lived impulse response to a TFP shock.

6 Discussion and Policy Implications

7 Conclusions

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8 Appendix

Figure 5: Domestic Expenditures and Factor Income

Figure 6: Output and Domestic Expenditures