The Government Intertemporal Budget Constraint and Ricardian Equivalence

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0.1 Government

Consider the following one period government budget,

$$B_{t+1} = (1+r)\,B_t + G_t - T_t \tag{1}$$

where B is the stock of bonds issued by the government (that is B is the stock of real debt - which is an asset in the hand of the private sector), and G, T and r have the usual meanings. Iterating forward one period we have:

$$B_{t+2} = (1+r)\,B_{t+1} + G_{t+1} - T_{t+1} \tag{2}$$

which we can substitute in (1) to get

$$\frac{B_{t+2} - G_{t+1} + T_{t+1}}{1+r} = (1+r)\,B_t + G_t - T_t \tag{3}$$

Rearranging to separate sources of revenues and debt accumulation from expenditure, we get

$$G_t + \frac{G_{t+1}}{1+r} = -\,(1+r)\,B_t + T_t + \frac{T_{t+1}}{1+r} + \frac{B_{t+2}}{1+r} \tag{4}$$

We can continue iterating forward one more period, i.e.:

$$B_{t+3} = (1+r)\,B_{t+2} + G_{t+2} - T_{t+2} \tag{5}$$

which, inserted in (4), will give:

$$G_t + \frac{G_{t+1}}{1+r} + \frac{G_{t+2}}{(1+r)^2} = -\,(1+r)\,B_t + T_t + \frac{T_{t+1}}{1+r} + \frac{T_{t+2}}{(1+r)^2} + \frac{B_{t+3}}{(1+r)^2} \tag{6}$$

Repeating the process for n periods gives:

$$\sum_{s=t}^{t+n}\left(\frac{1}{1+r}\right)^{s-t}(G_s) = -\,(1+r)\,B_t + \sum_{s=t}^{t+n}\left(\frac{1}{1+r}\right)^{s-t}(T_s) + \frac{B_{t+n}}{(1+r)^{t+n-1}} \tag{7}$$

In order for (7) to be an intertemporal budget constraint of the form used in the class presentation we need a further assumption about the limit of $\frac{B_{t+n}}{(1+r)^{t+n-1}}$ for n that goes to infinity

$$\lim_{n=\infty} \frac{B_{t+n}}{(1+r)^{t+n-1}} = 0 \tag{8}$$

This condition, called "No Ponzi Game Condition", rules out the possibility of an explosive accumulation of government debt . Imposing this condition on (7) and taking the limit for $n \to \infty$ gives:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (G_s) = -(1+r) B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (T_s) \tag{9}$$

which is the intertemporal budget constraint for the government. This methodology of finding the resources constraint in an intertemporal decision problem can be applied to any other agent in the economy and to the economy as a whole, relative to the rest of the world.

0.2 Consumer

A typical consumer budget is

$$A_{t+1} = (1+r) A_t + Y_t - C_t - T_t \tag{10}$$

where now A is stock of assests in the hand of the public. Iterating forward we have

$$A_{t+2} = (1+r) A_{t+1} + Y_{t+1} - C_{t+1} - T_{t+1} \tag{11}$$

which gives, as before,

$$C_t + \frac{C_{t+1}}{1+r} = (1+r) A_t + Y_t + \frac{Y_{t+1}}{1+r} + \frac{A_{t+2}}{1+r} \tag{12}$$

Iterating the process forward and imposing the condition that people cannot accumulate assets indefenetely we have

$$\lim_{n=\infty} \frac{A_{t+n}}{(1+r)^{t+n-1}} = 0 \tag{13}$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (C_s) = (1+r) A_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - T_s) \tag{14}$$

If the economy is closed and the only assets is riskless government bond, it is easy to show that government financing methods are irrelevant - Just substitute (10) into (15) and, knowing that $A = B$ you obtain:

$$
\begin{aligned}
\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s) &= (1+r)\,A_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s)-\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(T_s) \\
\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s) &= (1+r)\,A_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s)-\left[(1+r)\,B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(G_s)\right] \\
\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s) &= \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-G_s) \qquad (15)
\end{aligned}
$$

Equation (16) simply states that public debt is not part of consumer wealth!

0.3 Current Account

The same story could be replicated when considering the position of a country relative to the rest of the world. The current account identity operates as a budget constraint for the entire country

$$CA_t = Y_t + rF_t - G_t - C_t = NO_t + rF_t - C_t = F_{t+1} - F_t$$

which is a foregn assets accumulation equation. If the economy runs current account deficits, its citizens are borrowing from abroad ($\Delta F_{t+1} < 0$), and viceversa if $CA > 0$. Is there a budget constraint with the foreign sector too? Same procedure applies:

$$F_{t+1} = NO_{t+1} + (1+r)\,F_{t+1} - C_{t+1}$$

Integrating forward as before we have:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s) = (1+r)\,F_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(NO_s)$$