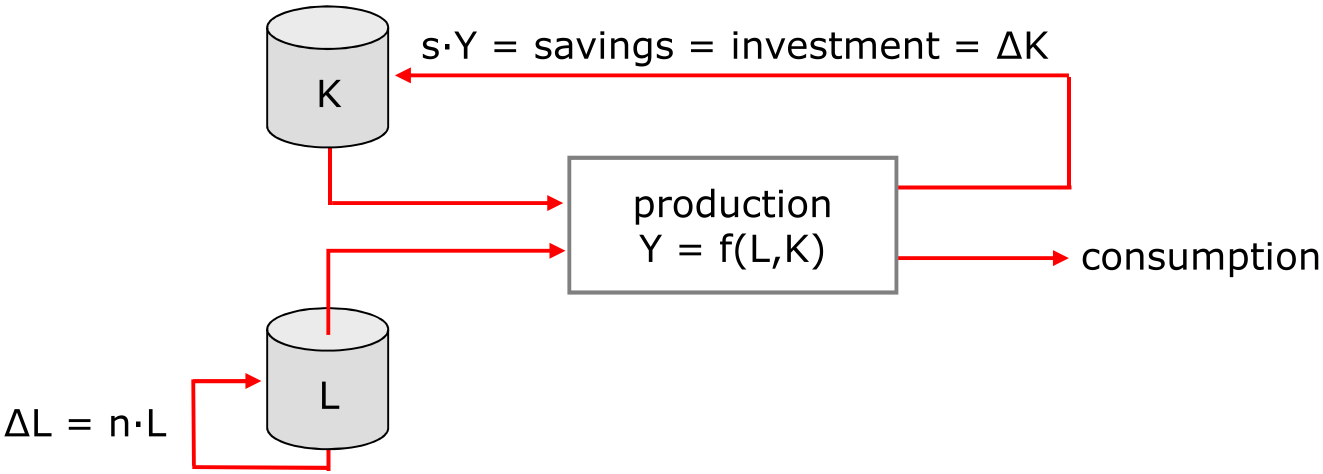
# Macroeconomics

## Review of Growth Theory
## Solow and the Rest

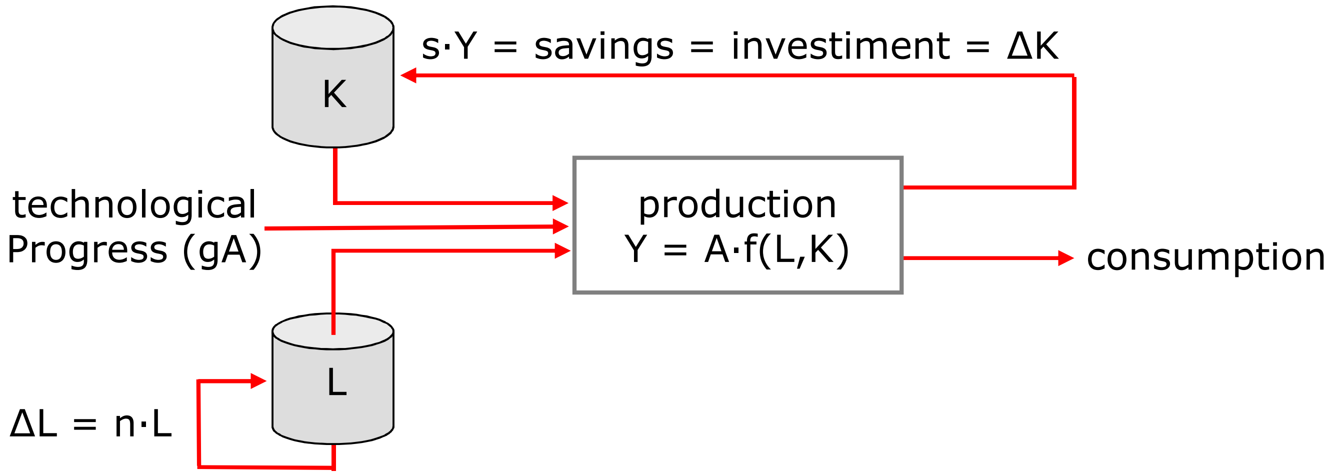
# Basic Neoclassical Growth Model




- exogenous population (labor) growth rate *n*
- saving rate: exogenous or derived via dynamic (= intertemporal) optimization (rate of time preference)

- **gY = n and gy = 0**

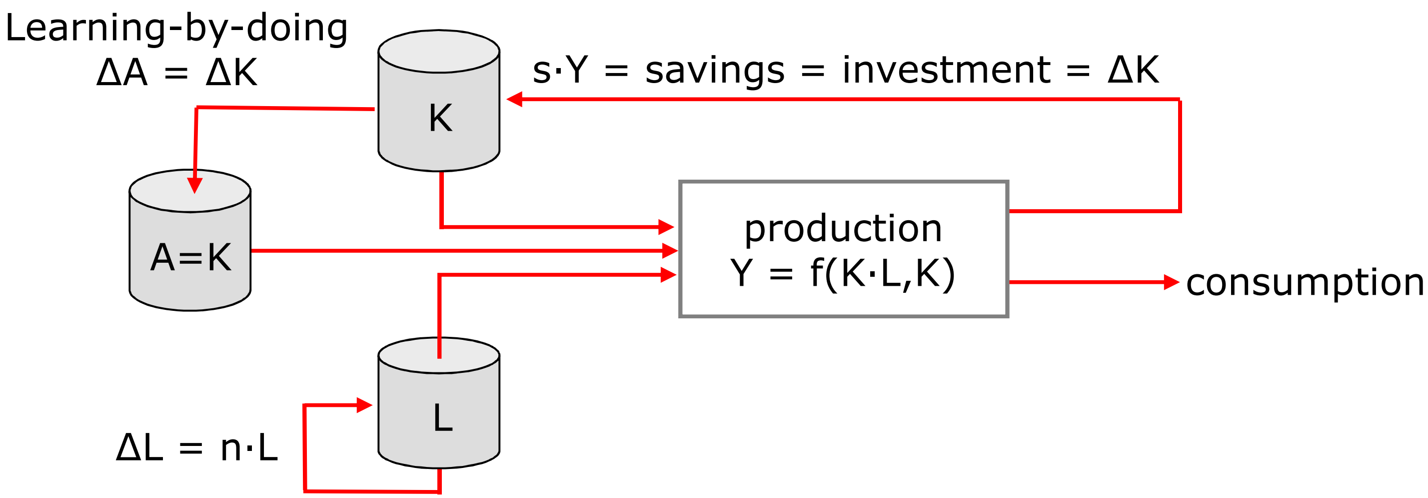
# Effect of Technological Progress




- exogenous, disembodied, factor augmenting technological progress
- technological progress does not consume any resources but is “produced by time” (like manna from heaven)

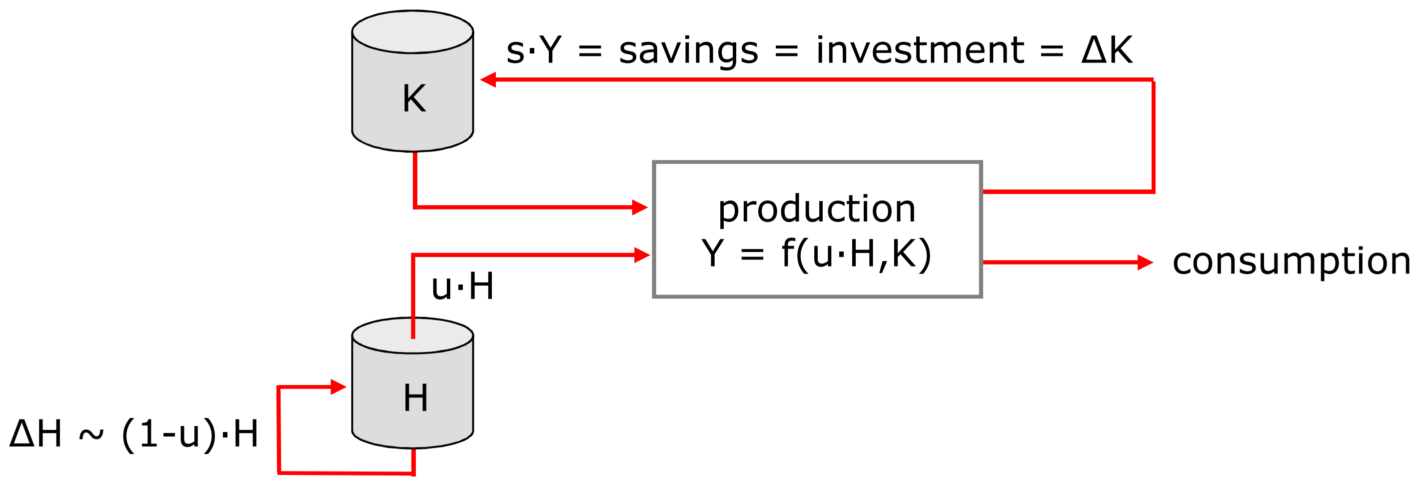
- **$gY = n + gA$ and $gy = gA$**

# Human Capital: Learning-by-Doing / AK Model



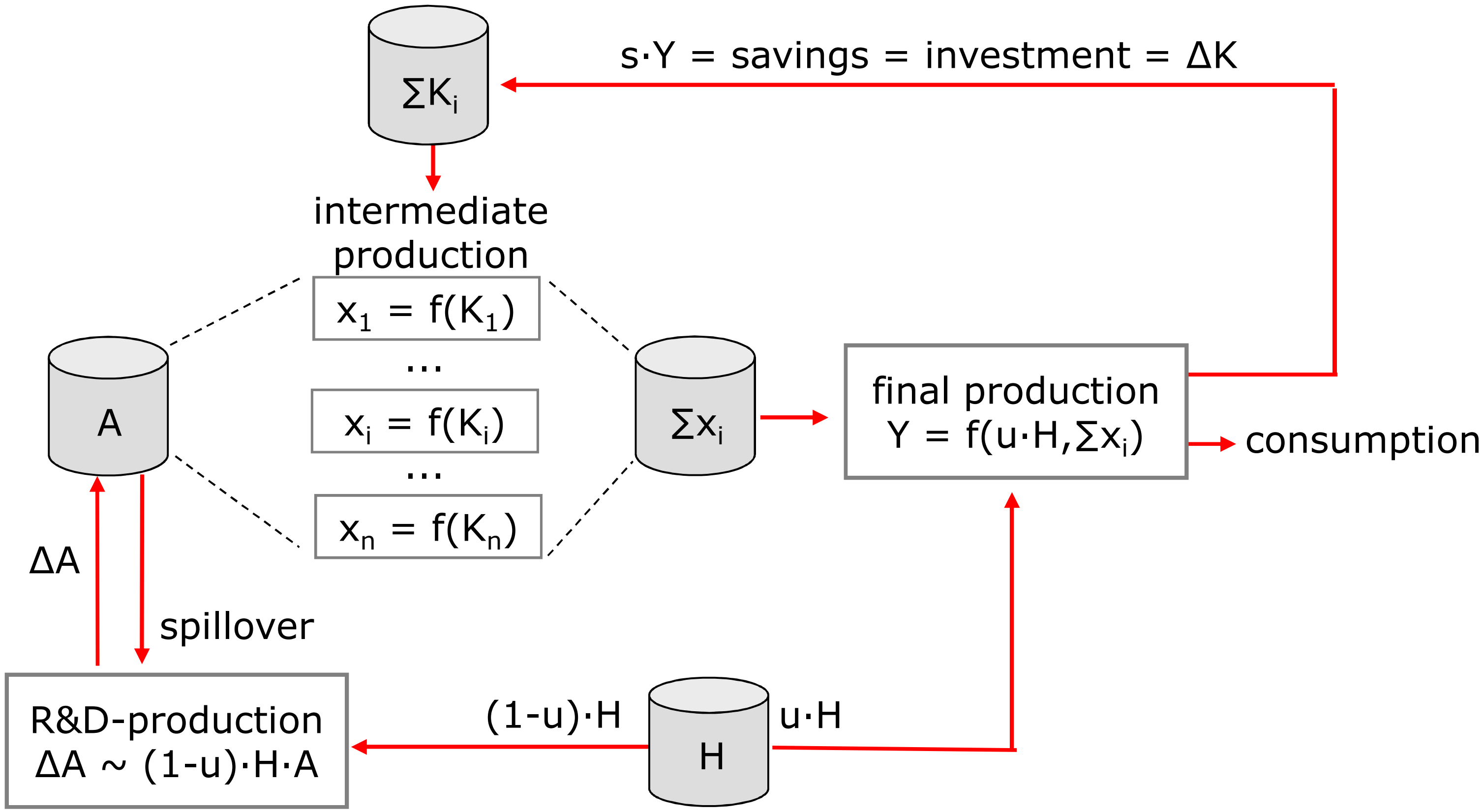

- labor augmenting technological progress with capital stock as a proxy for accumulated experience (know-how)
- growth engine: production becomes proportional to capital stock (no more diminishing returns to scale!)
- steady-state growth rate influenced by: saving rate (rate of time preference), country size, technology
- know-how as a public good (externality)
  ⇨ inefficient steady-state growth rate
  ⇨ suggests public subsidies for investments

# Human Capital Accumulation




- labor-embodied know-how
- growth engine: no diminishing returns to scale in human capital formation ($\Delta H \sim H$)
- steady-state growth rate influenced by: saving rate, productivity of the education sector
- efficient steady-state growth rate (optimal allocation of resources via market processes)

# Producing Technological Progress via Research & Development



# Producing Technological Progress via Research & Development

- reasearch done by rational, profit-maximizing agents
- (old) idea: growth is sustained by increased specialization of labor
- incentives to innovate (monopoly profits) stem from imperfect competition in the intermediate sector (patent protection)
- growth engine: spillovers and specialized capital
- steady-state growth rate influenced by:
  - productivity in the R&D-sector
  - stock of human capital
  - saving rate
  - productivity in the final goods sector (negative!)
- inefficient steady-state growth rate (too low) due to spillover-externalities

# Models of Economic Growth

- Neoclassical growth models

- Endogenous growth models

# Neoclassical growth model

- Model growth of GDP per worker via capital accumulation
- Key elements:
  - **Production function** (GDP depends on technology, labour and physical capital)
  - **Capital accumulation equation** (change in net capital stock equals gross investment [=savings] less depreciation).
- Questions:
  - how does capital accumulation (net investment) affect growth?
  - what is role of savings, depreciation and population growth?
  - what is role of technology?

# Solow-Swan equations

$Y = Af(K, L)$ (production function)

$Y = \text{GDP}, A = \text{technology},$

$K = \text{capital}, L = \text{labour}$

$\frac{dK}{dt} = sY - \delta K$ (capital accumulation equation)

$s = \text{proportion of GDP saved } (0 < s < 1)$

$\delta = \text{depreciation rate (as proportion) } (0 < \delta < 1)$

Solow-Swan analyse how these two equations interact.

*Y* and *K* are endogenous variables; *s*, $\delta$ and growth rate of *L* and/or *A* are exogenous (parameters).

Outcome depends on the **exact** functional form of production function and parameter values.

# Neoclassical production functions

Solow-Swan assume:

a) diminishing returns to capital or labour (the 'law' of diminishing returns), and

b) constant returns to scale (e.g. doubling *K* and *L*, doubles *Y*).

For example, the Cobb-Douglas production function

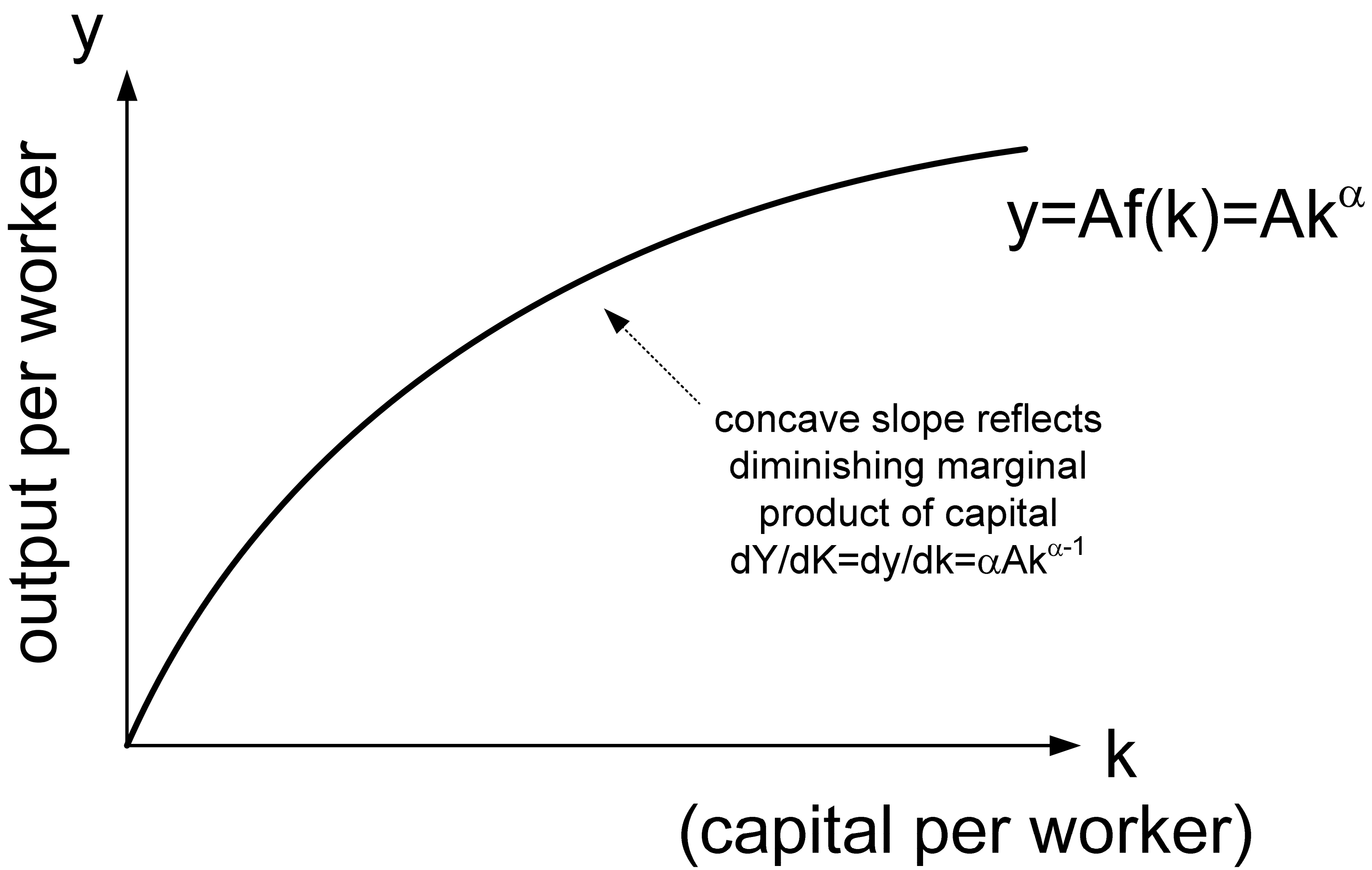
$$Y = AK^{\alpha}L^{1-\alpha} \qquad \text{where } 0 < \alpha < 1$$

$$y = \frac{Y}{L} = \frac{AK^{\alpha}L^{1-\alpha}}{L} = \frac{AK^{\alpha}}{L^{\alpha}} = A\left(\frac{K}{L}\right)^{\alpha} = Ak^{\alpha}$$

Hence, now have *y* = output (GDP) per worker as function of capital to labour ratio (*k*)

# GDP per worker and *k*

Assume *A* and *L* constant (no technology growth or labour force growth)



# Accumulation equation

If *A* and *L* constant, can show*

$$\frac{dk}{dt} = sy - \delta k$$

This is a differential equation. In words, the change in capital to labour ratio over time = investment (saving) per worker <u>*minus*</u> depreciation per worker.

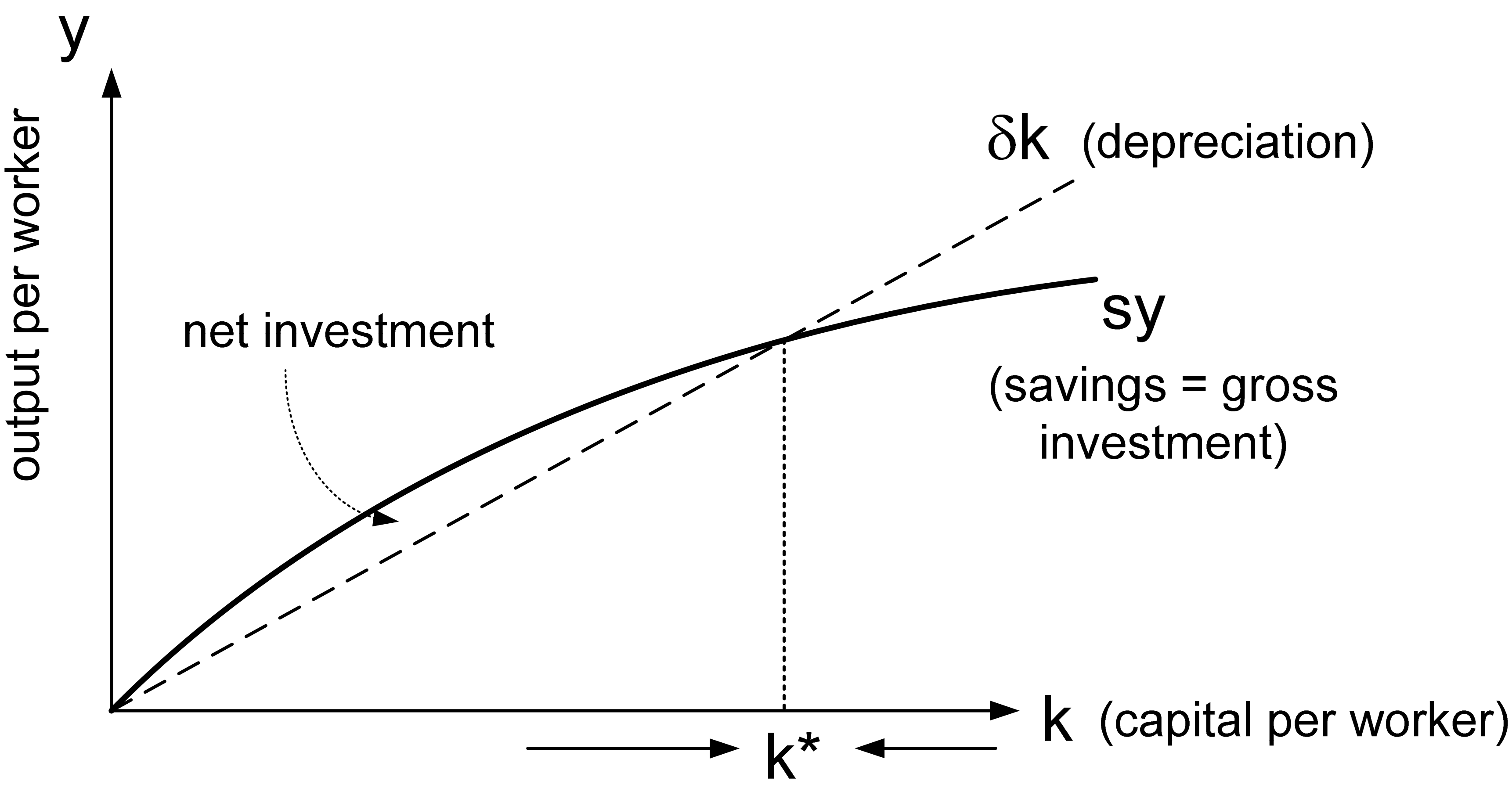
Any positive change in *k* will increase *y* and generate economic growth. Growth will stop if *dk/dt=0.*

*accumulation equation is: $\frac{dK}{dt} = sY - \delta K$, divide by L yields $\frac{dK}{dt} / L = sy - \delta k$

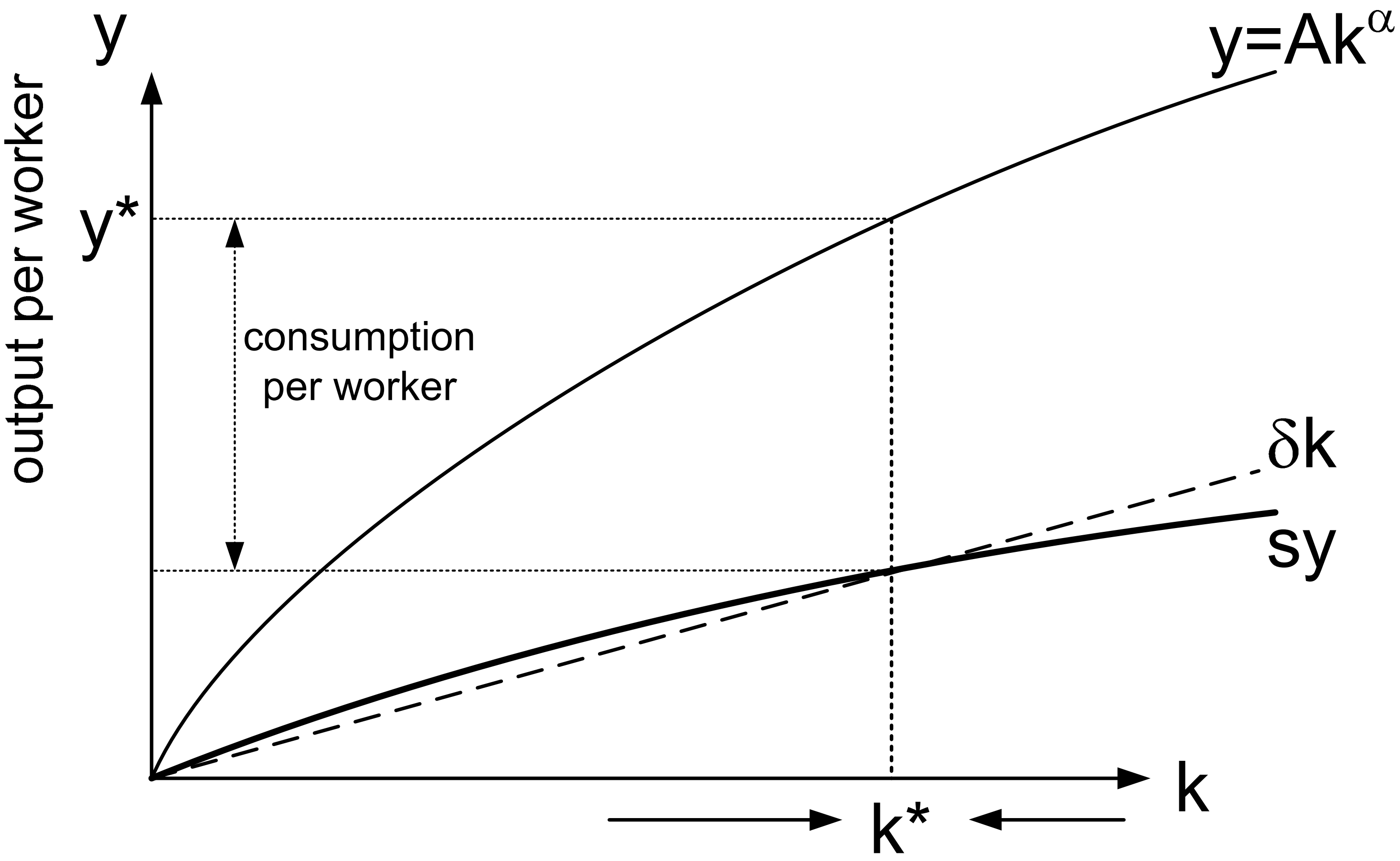
Also note that, $\frac{dk}{dt} = d\left(\frac{K}{L}\right) / dt = \frac{dK}{dt} / L$ since $L$ is a constant.

# Graphical analysis of $\frac{dk}{dt} = sy - \delta k$

(Note: *s* and $\delta$ constants)



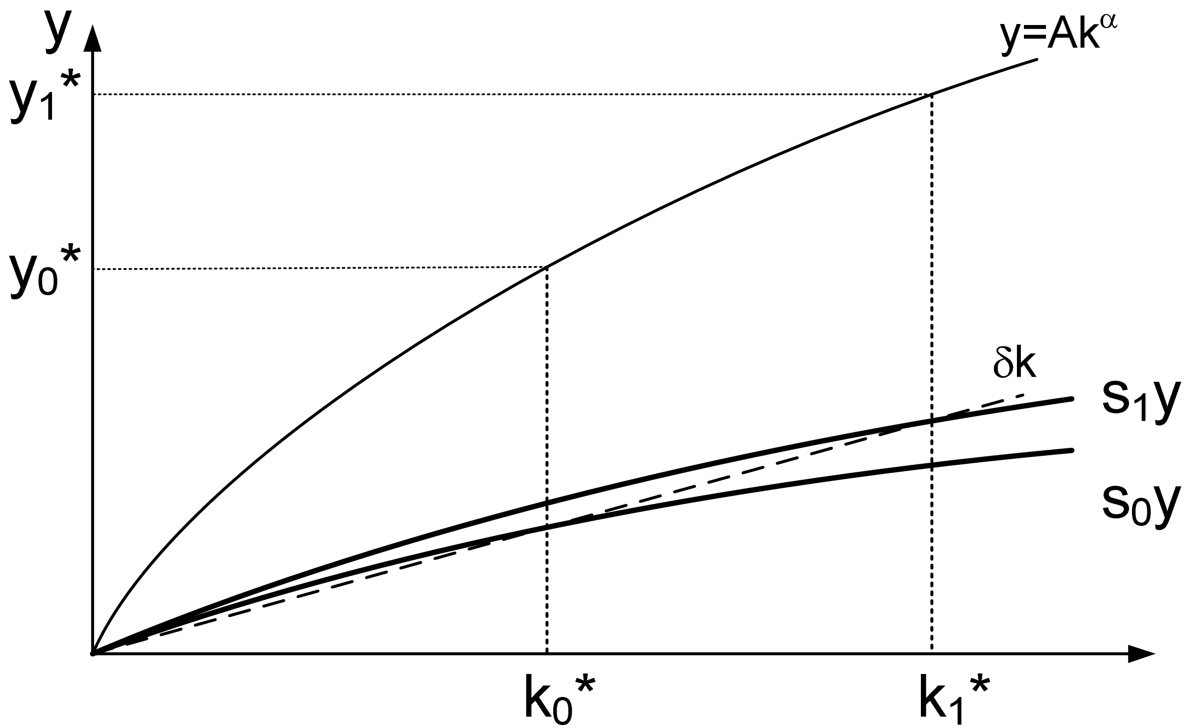
# Solow-Swan equilibrium




GDP p.w. converges to $y^* = A(k^*)^\alpha$. If A (technology) and L constant, y* is also constant: **no long run growth**.

# What happens if savings increased?

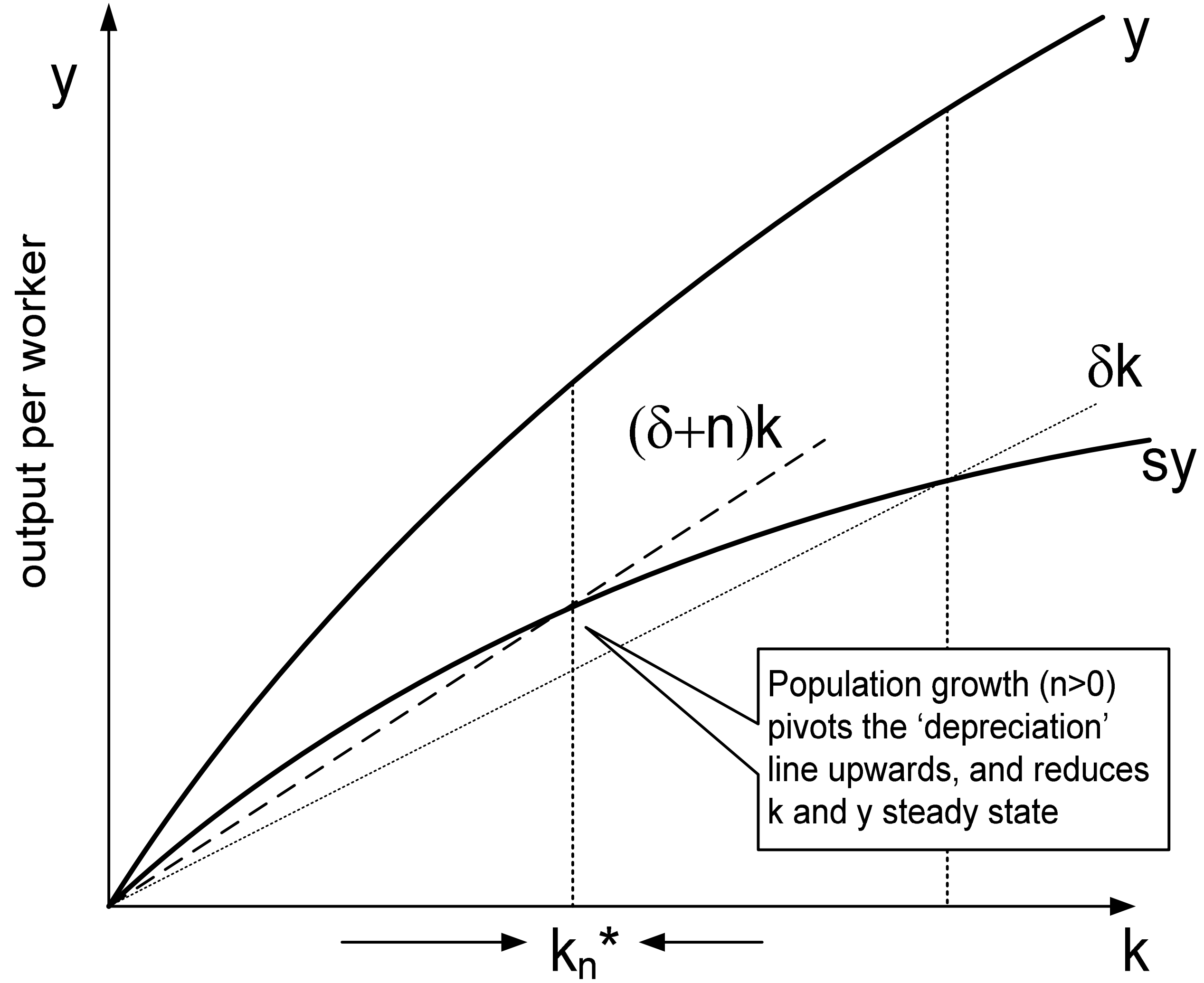
- raising saving increases *k** and *y**, but long run growth still zero (e.g. $s_1 > s_0$ below)
- call this a "levels effect"
- growth increases in short run (as economy moves to new steady state), but no permanent 'growth effect'.



# What if labour force grows?

Accumulation eqn now $\frac{dk}{dt} = sy - (\delta + n)k$ where $n = \frac{dL}{dt} / L$ (math note 2)

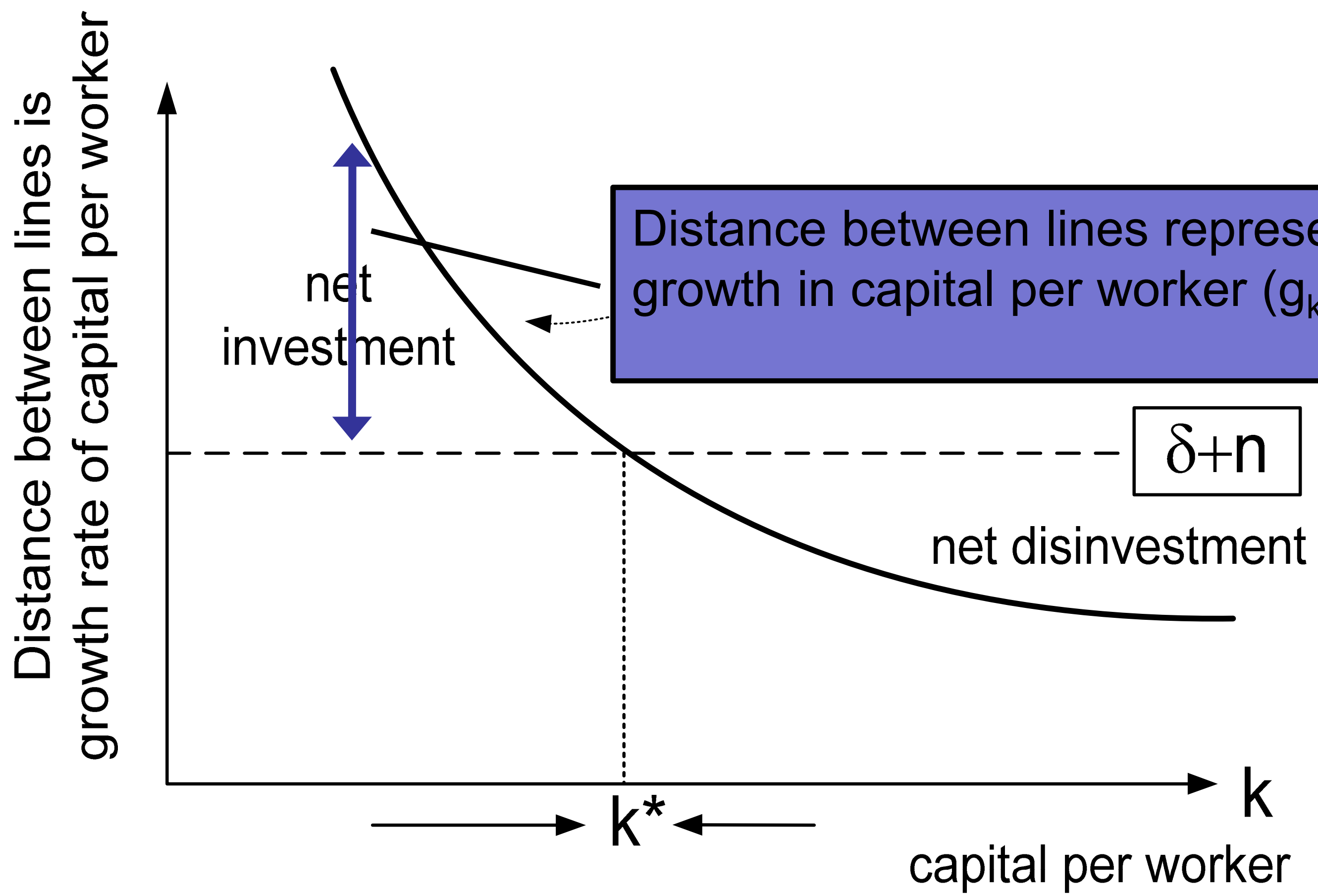
Population growth reduces equilibrium level of GDP per worker (but **long run growth still zero**) if technology static



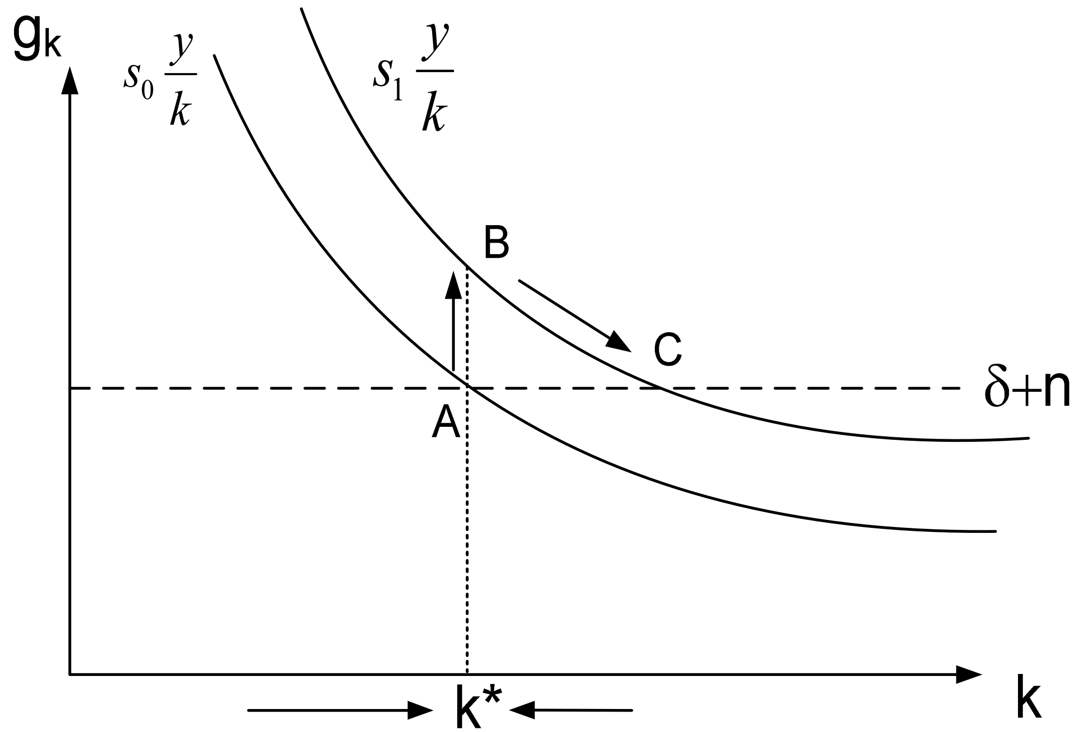
# Analysis in growth rates

Can illustrate above with graph of $g_k$ and k

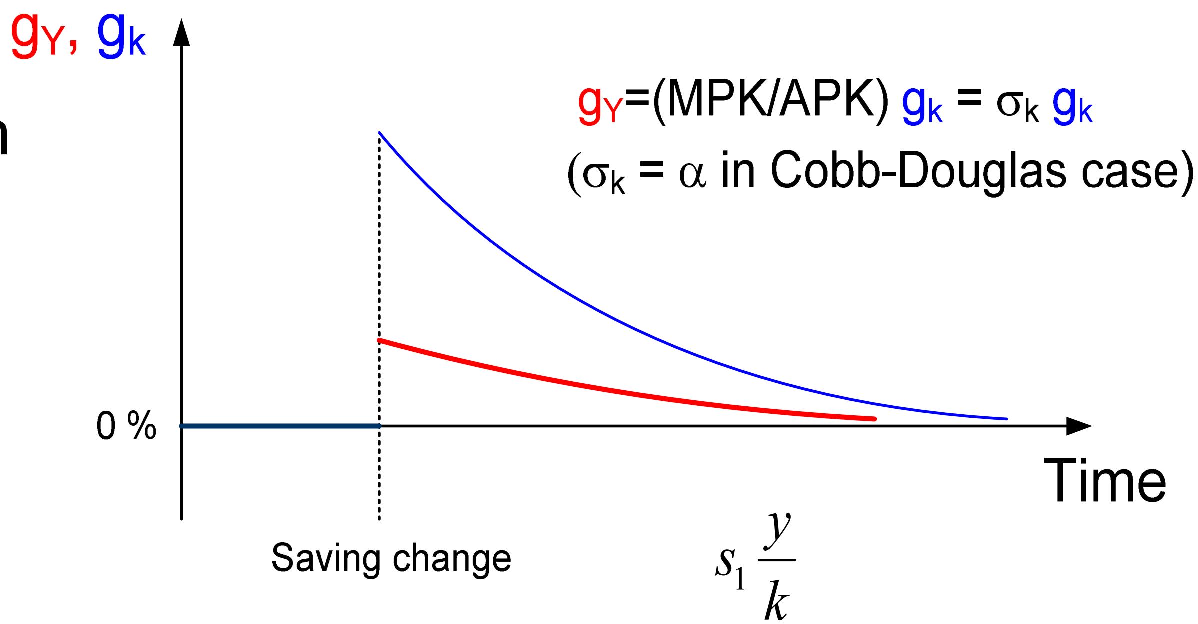
$$\frac{dk}{dt} = sy - (\delta + n)k \Rightarrow \frac{dk/dt}{k} = g_k = s\frac{y}{k} - (\delta + n)$$



# Rise in savings rate ($s_0$ to $s_1$)




NB: This graph of how growth rates change over time



# Golden rule

- The 'golden rule' is the 'optimal' saving rate ($s_G$) that maximises consumption per head.
- Assume *A* is constant, but population growth is *n*.
- Can show that this occurs where the marginal product of capital equals $(\delta + n)$

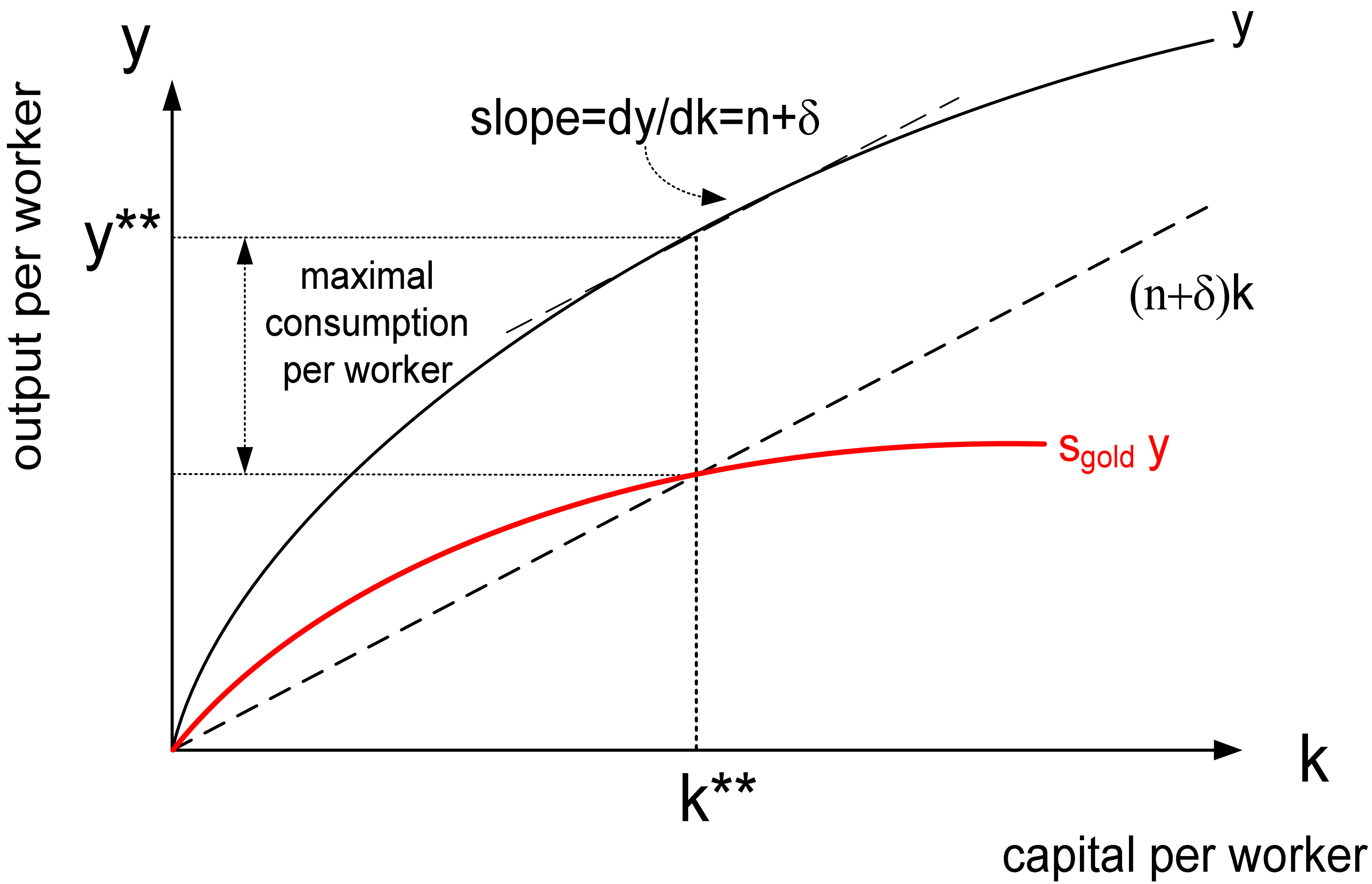
Proof: $\frac{dk}{dt} = sy - (\delta + n)k = 0$ at steady state,

hence $sy^* = (\delta + n)k^*$, where * indicates steady state equilibrium value

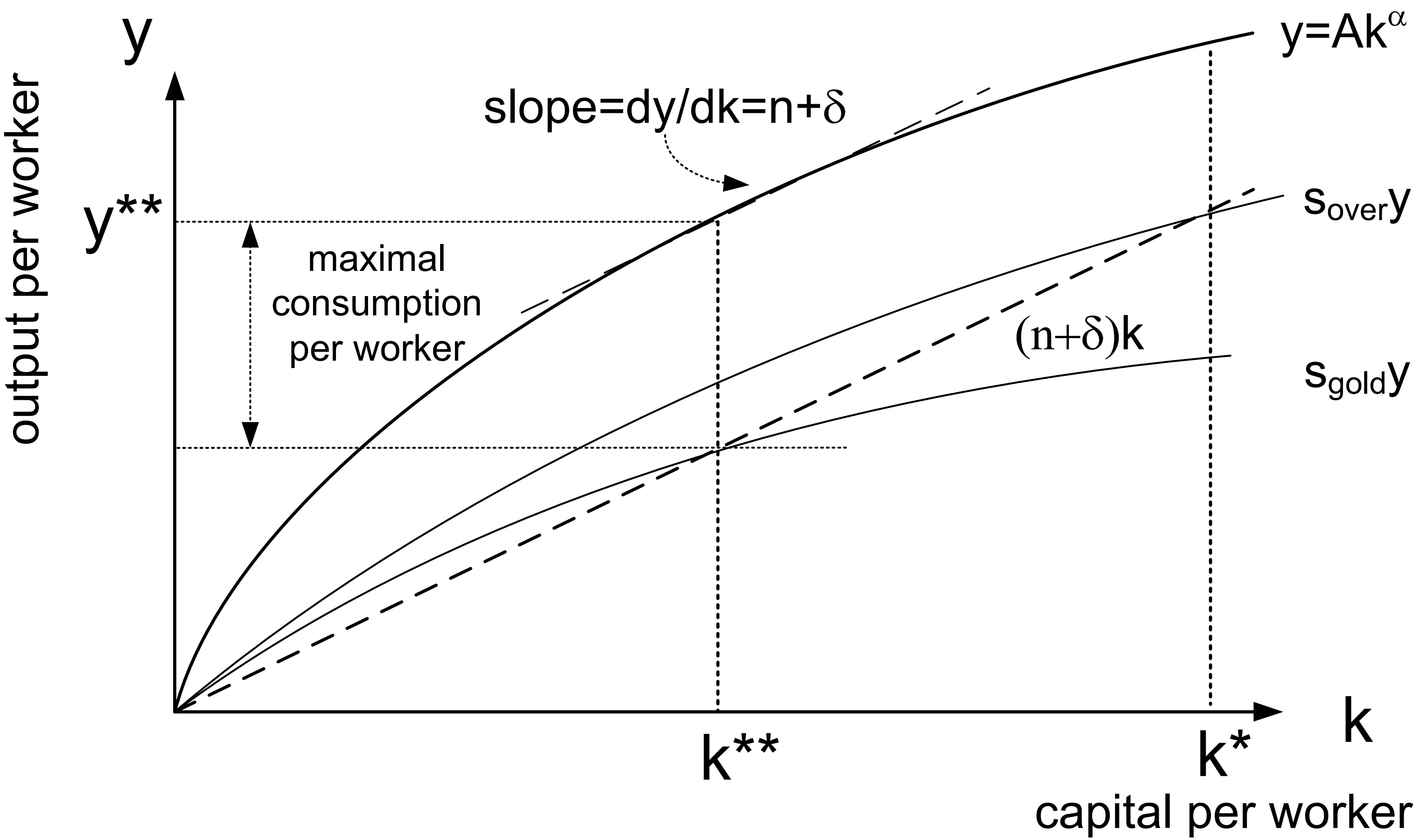
The problem is to: $\max_k c = y - sy = y^* - (\delta + n)k^*$

First order condition: $0 = \frac{dy^*}{dk^*} - (\delta + n)$ hence $MP_k = \frac{dy^*}{dk^*} = \delta + n$

# Graphically find the maximal distance between two lines



# … over saving




Economies can **over save**. Higher saving does increase GDP per worker, but real objective is consumption per worker.

# Golden rule for Cobb Douglas case

- $Y=K^{\alpha}L^{1-\alpha}$ or $y = k^{\alpha}$
- Golden rule states: $MP_k = \alpha(k^*)^{\alpha-1} = (n + \delta)$
- Steady state is where: $sy^* = (\delta + n)k^*$
- Hence, $sy^* = [\alpha(k^*)^{\alpha-1}]k^*$

  or $s = \alpha(k^*)^{\alpha} / y^* = \alpha$

  Golden rule saving ratio = $\alpha$ for $Y=K^{\alpha}L^{1-\alpha}$ case

# Solow's surprise*

- Solow's model states that investment in capital cannot drive **long run** growth in GDP per worker
- Need technological change (growth in A) to avoid **diminishing returns to capital**
- Easterly (2001) argues that "capital fundamentalism" view widely held in World Bank/IMF from 60s to 90s, despite lessons of Solow model
- Policy lesson: don't advise poor countries to invest without due regard for technology and incentives

# What if technology ($A$) grows?

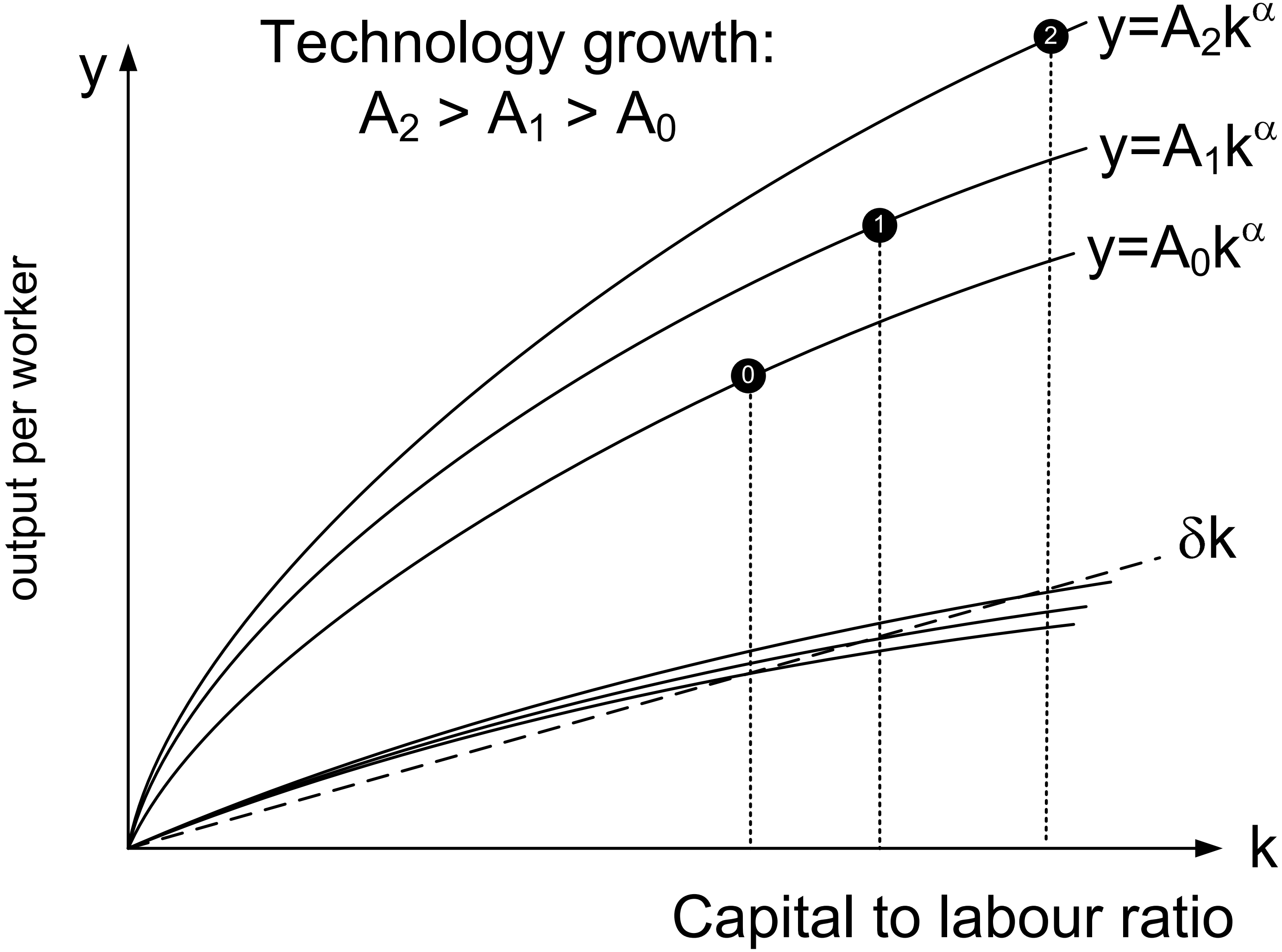
- Consider $y=Ak^{\alpha}$, and $sy=sAk^{\alpha}$, these imply that output can go on increasing.
- Consider marginal product of capital ($MP_k$)

$$MP_k = dy/dk = \alpha A k^{\alpha-1},$$

if $A$ increases then $MP_k$ can keep increasing (no 'diminishing returns' to capital)

- implies **positive long run growth**

# …. graphically, the production function simply shifts up



# …. mathematically

Easier to use $Y = K^{\alpha}(AL)^{1-\alpha}$ where $0 < \alpha < 1$

(This assumes A augments labour (Harrod-neutral technological change)

Can re-write $K^{\alpha}(AL)^{1-\alpha} = A^{1-\alpha}K^{\alpha}L^{1-\alpha}$

Assume $\frac{\mathrm{dA}}{\mathrm{dt}} / A = g_A$ (for reference this same as $\mathrm{A_t} = \mathrm{A_o}e^{g_A t}$)

Trick to solving is to re-write as

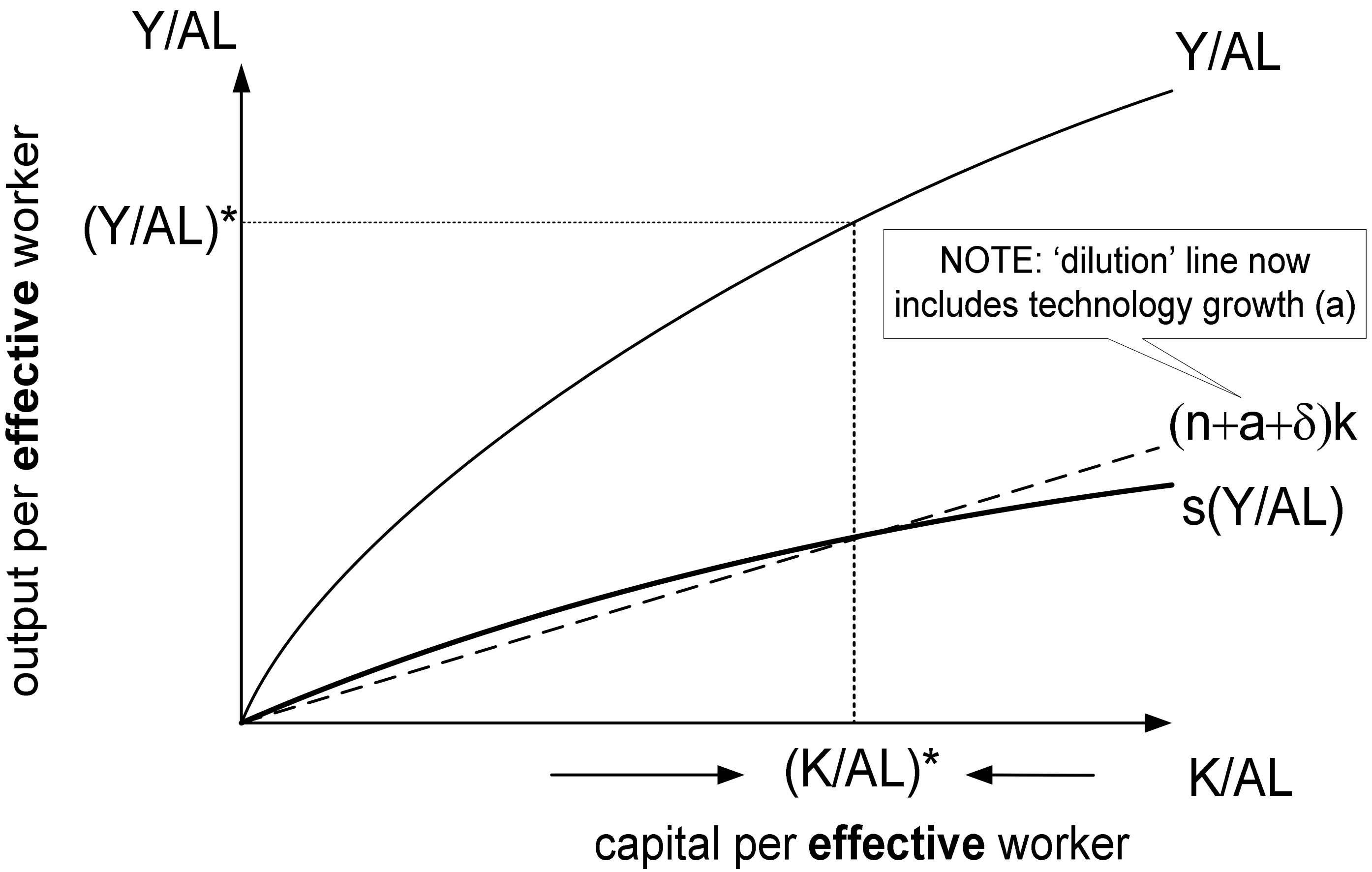
$$\tilde{y} = \frac{Y}{AL} = \frac{K^{\alpha}(AL)^{1-\alpha}}{AL} = \left(\frac{K}{AL}\right)^{\alpha} = (\tilde{k})^{\alpha}$$

where $\tilde{y}$=output per 'effective worker', and $\tilde{k}$ = capital per 'effective worker'

Can show $\frac{d\tilde{k}}{dt} / \tilde{k} = s(\tilde{k})^{\alpha} - (n + a + \delta)\tilde{k}$

This can be solved (plotted) as in simpler Solow model.

# Output (capital) per effective worker diagram




If *Y/AL* is a constant, the growth of Y must equal the growth rate of *L* plus growth rate of *A* (i.e. *n*+*a*)

And, **growth in <u>GDP per worker</u> must equal growth in *A***.

# Summary of Solow-Swan

- Solow-Swan, or neoclassical, growth model, implies countries converge to steady state GDP per worker (if **no growth** in technology)
- if countries have same steady states, poorer countries grow faster and ‘converge’
  - call this classical convergence or ‘convergence to steady state in Solow model’
- changes in savings ratio causes “level effect”, but no **long run** growth effect
- higher labour force growth, ceteris paribus, implies lower GDP per worker
- Golden rule: economies can over- or under-save (note: can model savings as endogenous)

# Technicalities of Solow-Swan

- Textbooks (Jones 1998, and Carlin and Soskice 2006) give full treatment, in short:
- **Inada conditions** needed ( “growth will start, growth will stop”)

$$\lim_{K \to \infty} \frac{dY}{dK} = 0, \qquad \lim_{K \to 0} \frac{dY}{dK} = \infty,$$

- It is possible to have production function where d*Y*/d*K* declines to positive constant (so growth declines but never reaches zero)
- Exact outcome of Solow model does depend on precise functional forms and parameter values
- BUT, with standard production function (Cobb-Douglas) Solow model predicts economy moves to steady state because of diminishing returns to capital (assuming no growth in technology *A*)

# Endnotes

Math note 1: $y_t = y_0 e^{gt}$ can be used to analyse impact of growth over time

Let y=GDP p.w., g=growth (e.g. $0.02 \equiv 2\%$), t=time.

Hence, for $g = 0.02$ and $t = 100$, $y_t / y_0 = e^2 = 7.39$

Math Note 2:

Start with $\frac{dK}{dt} = sY - \delta K$, divide by $L$ yields $\frac{dK}{dt} / L = sy - \delta k$

Note that $\frac{dk}{dt} = d\left(\frac{K}{L}\right) / dt = \left[\frac{dK}{dt} L - \frac{dL}{dt} K\right] / L^2$ (quotient rule)

simplify to $\frac{dK}{dt} / L - \left(\frac{dL}{dt} / L\right)\frac{K}{L}$ or $\frac{dK}{dt} / L - nk$ (since $n$ is labour growth and $K / L = k$)

hence $\frac{dk}{dt} + nk = \frac{dK}{dt} / L = sy - \delta k$

hence $\frac{dk}{dt} = sy - (\delta + n)k$

# Questions for discussion

1. What is the importance of diminishing marginal returns in the neoclassical model? How do other models deal with the possibility of diminishing returns?
2. Explain the effect of (i) an increase in savings ratio (ii) a rise in population growth and (iii) an increase in exogenous technology growth in the neoclassical model.
3. What is the golden rule? Can you think of any countries that have broken the golden rule?

# Growth accounting

$$Y_t = B_t K_t^{\alpha} L_t^{1-\alpha} \Rightarrow$$

$$\ln Y_T = \ln B_T + \alpha \ln K_T + \left(1-\alpha\right)\ln L_T \text{ and}$$

$$\ln Y_t = \ln B_t + \alpha \ln K_t + \left(1-\alpha\right)\ln L_t \Rightarrow$$

$$\frac{\ln Y_T - \ln Y_t}{T-t} = \frac{\ln B_T - \ln B_t}{T-t} + \alpha \frac{\ln K_T - \ln K_t}{T-t} + \left(1-\alpha\right)\frac{\ln L_T - \ln L_t}{T-t} \Leftrightarrow$$

$$g_{T,t}^{Y} = g_{T,t}^{B} + g_{T,t}^{K} + \left(1-\alpha\right) g_{T,t}^{L}.$$

- With data for $Y_\tau, K_\tau$ and $L_\tau$, $\tau = t, T$ and with $\alpha = 1/3$ we can compute $g_{T,t}^{B}$ as a residual. We call this the **Solow residual**.

- Why not growth accounting in levels?

# Growth accounting per capita

$$Y_t = B_t K_t^{\alpha} L_t^{1-\alpha} \Rightarrow y_t = B_t k_t^{\alpha}$$

$$\ln y_T = \ln B_T + \alpha \ln k_T \quad \text{and} \quad \ln y_t = \ln B_t + \alpha \ln k_t \Rightarrow$$

$$\frac{\ln y_T - \ln y_t}{T-t} = \frac{\ln B_T - \ln B_t}{T-t} + \alpha \frac{\ln k_T - \ln k_t}{T-t} \Leftrightarrow$$

$$g_{T,t}^{y} = g_{T,t}^{B} + g_{T,t}^{k}$$

- With data for $y_\tau$ and $k_\tau$ and with $\alpha = 1/3$ we can once more compute the Solow residual, $g_{T,t}^{B}$.
- We can use this residual to check the underlying ”technological growth”

# Growth Accounting in South Africa

| Number | Period | Output growth | Capital contribution | Labour contribution | TFP |
|---|---|---|---|---|---|
| Without Human Capital | 1985-1994<br>1995-2004 | 0.8<br>3.0 | 0.45<br>0.62 | 0.63<br>0.62 | -0.28<br>1.76 |
| Labor adjusted by years of schooling | 1985-1994<br>1995-2004 | 0.8<br>3.0 | 0.45<br>0.62 | 1.11<br>0.88 | -0.76<br>1.50 |
| Labor adjusted by skill level | 1985-1994<br>1995-2004 | 0.8<br>3.0 | 0.45<br>0.62 | 1.49<br>0.95 | -1.14<br>1.43 |

# Endogenous Growth Models

# Endogenous growth models - topics

- Recap on growth of technology (A) in Solow model (…..does allow long run growth)
- Endogenous growth models
- Non-diminishing returns to ‘capital’
- Role of human capital
- Creative destruction models
- Competition and growth
- Scale effects on growth

# Exogenous technology growth

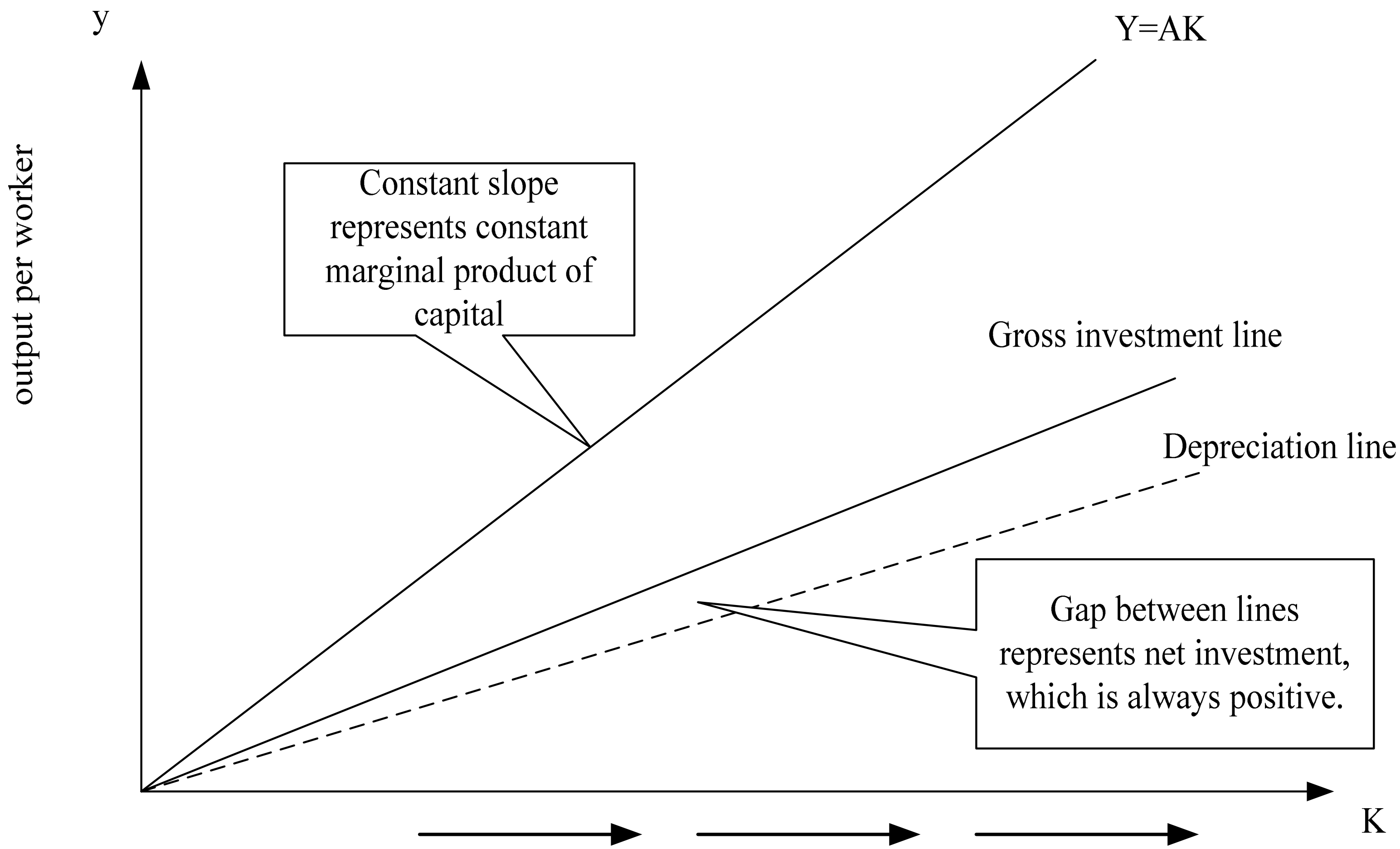
- Solow (and Swan) models show that technological change drives growth
- But growth of technology is not determined within the model (it is **exogenous**)
- Note that it does not show that capital investment is unimportant ( $A\uparrow \Rightarrow \uparrow y$ and $\uparrow MP_k$, hence $\uparrow k$)
- In words …. better technology raises output, but also creates new capital investment opportunities
- Endogenous growth models try to make **endogenous** the driving force(s) of growth
- Can be technology or other factors like learning by workers

# The AK model

- The '*AK* model' is sometimes termed an 'endogenous growth model'
- The model has $Y = AK$

  where *K* can be thought of as some composite 'capital and labour' input
- Clearly this has **constant marginal product of capital** ($MP_k$ = dY/dK=A), hence long run growth is possible
- Thus, the 'AK model' is a simple way of illustrating endogenous growth concept
- However, it is very simple! 'A' is poorly defined, yet critical to growth rate
- Also composite 'K' is unappealing

# The AK model in a diagram



# Endogenous technology growth

- Suppose that technology depends on past investment (i.e. the process of investment generates new ideas, knowledge and learning).

$$A = g(K) \qquad \text{where} \quad \frac{dA}{dK} > 0$$

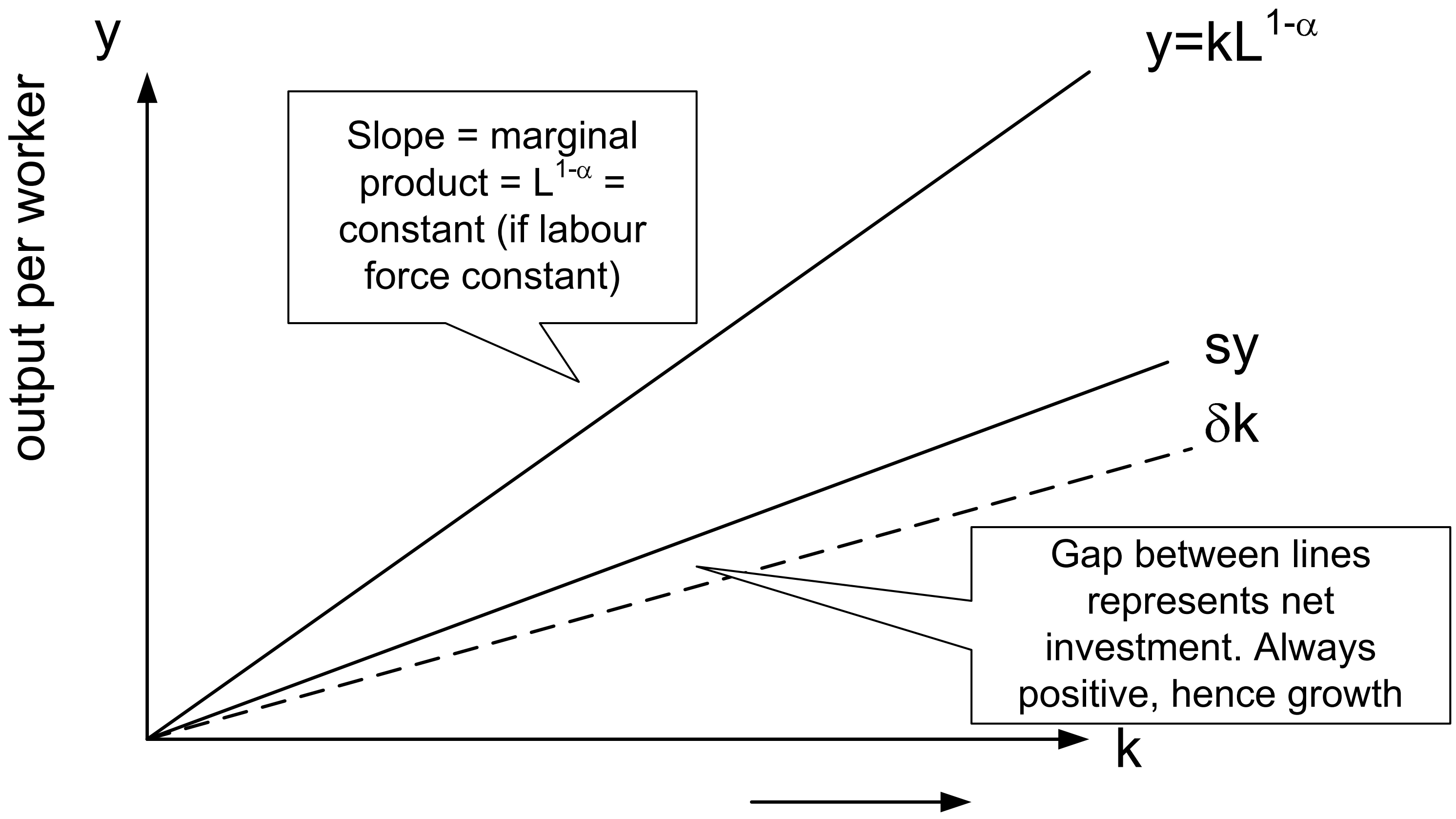
Specifically, let $A = K^{\beta}$ $\qquad \beta > 0$

Cobb-Douglas production function

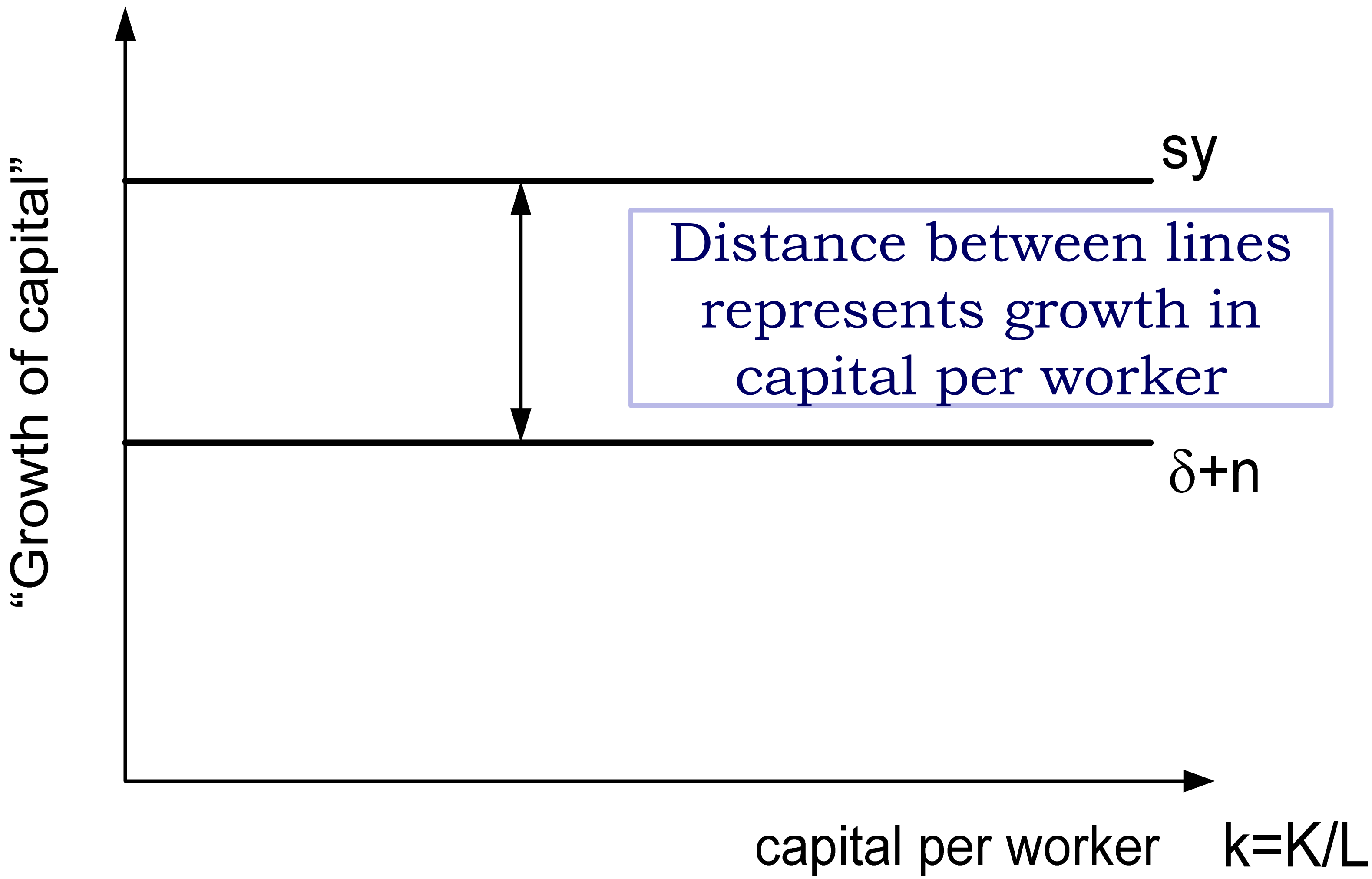
$$Y = AK^{\alpha}L^{1-\alpha} = [K^{\beta}]K^{\alpha}L^{1-\alpha} = K^{\alpha+\beta}L^{1-\alpha}$$

If $\alpha+\beta = 1$ then **marginal product of capital is constant** ($dY/dK = L^{1-\alpha}$).

- Assuming $A$=g($K$) is Ken Arrow’s (1962) learning-by-doing paper
- Intuition is that learning about technology prevents marginal product declining



# Situation on growth diagram



# Increasing returns to scale

$$Y = K^{\alpha+\beta} L^{1-\alpha} \quad \text{with } \alpha + \beta = 1$$

- "Problem" with $Y = K^1 L^{1-\alpha}$ is that it exhibits **increasing returns to scale** (doubling $K$ and $L$, more than doubles $Y$)
- IRS $\Rightarrow$ large firms dominate, no perfect competition (no P=MC, no first welfare theorem, …..)
- …. solution, assume feedback from investment to $A$ is <u>external</u> to firms (note this is positive externality, or spillover, from microeconomics)

# Knowledge externalities

A firm's production function is $Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$

but $A_i$ depends on aggregate capital

(hence firm does not 'control' increasing returns)

- Romer (1986) paper formally proves such a model has a competitive equilibrium
- However, the importance of externalities in knowledge (R&D, technology) long recognised
- Endogenous growth theory combines IRS, knowledge externalities and competitive behaviour in (dynamic optimising) models

# More formal endogenous growth models

- Romer (1990), Jones (1995) and others use a model of profit-seeking firms investing in R&D
- A firm's R&D raises its profits, but also has a **positive externality** on other firms' R&D productivity (can have competitive behaviour at firm-level, but IRS overall)
- Assume $\boldsymbol{Y=K^{\alpha}(AL_Y)^{1-\alpha}}$
- Labour used either to produce output ($L_Y$) or technology ($L_A$)
- As before, $A$ is technology (also called 'ideas' or 'knowledge')
- Note total labour supply is $L = L_Y + L_A$

# Romer model

Assume $\frac{dA}{dt} = \delta L_A^{\lambda} A^{\phi}$ $\delta > 0$

This is differential equation. Can $A$ have constant growth rate?

Answer: depends on parameters $\phi$ and $\lambda$ and growth of $L_A$

Romer (1990) assumed: $\lambda = 1, \phi = 1$

hence $\frac{dA}{dt} = \delta L_A A$

$\Rightarrow \frac{dA}{dt} / A = \delta L_A$ (>0 if some labour allocated to research)

If A has positive growth, this will give long run growth in GDP*p.w.*

Note that there is a 'scale effect' from $L_A$

Note '**knife edge**' property of $\phi$=1. If $\phi$>1, growth rate will accelerate over time; if $\phi$<1, growth rate falls.

# Jones model (semi-endogenous)

$\lambda > 0, \phi < 1$ (Jones, 1995)

Now $\frac{dA}{dt} = \delta L_A^{\lambda} A^{\phi} \quad \Rightarrow \quad \frac{dA}{dt} / A = \frac{\dot{A}}{A} = \frac{\delta L_A^{\lambda} A^{\phi}}{A} = \frac{\delta L_A^{\lambda}}{A^{1-\phi}}$

Can only have positive long run growth if far right term is constant

This only when $\lambda \frac{\dot{L}_A}{L_A} = (1-\phi)\frac{\dot{A}}{A}$ or $\frac{\dot{A}}{A} = \frac{\lambda}{(1-\phi)} \frac{\dot{L}_A}{L_A}$

In words: growth of technology = constant × labour growth

- No scale effects, no 'knife edge' property, but requires (exogenous) labour force growth hence "semi-endogenous" (see Jones (1999) for discussion)

# Human capital – the Lucas model

- Lucas defines human capital as the skill embodied in workers
- Constant number of workers in economy is N
- Each one has a human capital level of h
- Human capital can be used either to produce output (proportion *u*)
- Or to accumulate new human capital (proportion *1-u*)
- Human capital grows at a constant rate

$$dh/dt = h(1-u)$$

# Lucas model in detail

- The production of output (Y) is given by

  $$Y = AK^{\alpha} (uhN)^{1-\alpha} h_a^{\gamma}$$

  where $0 < \alpha < 1$ and $\gamma \geq 0$

- Lucas assumed that technology (A) was constant

- Note the presence of the extra term $h_a^{\gamma}$ - this is defined as the 'average human capital level'

- This allows for external effect of human capital that can also influence other firms, e.g. higher average skills allow workers to communicate better

- Main driver of growth - As h grows the effect is to scale up the input of workers N, so raising output Y and raising marginal product of capital K

# Creative destruction and firm-level activity

- many endogenous growth models assume profit-seeking firms invest in R&D (ideas, knowledge)
  - **Incentives**: expected **monopoly** profits on new product or process. This depends on probability of inventing and, if successful, expected length of monopoly (strength of intellectual property rights e.g. patents)
  - **Cost**: expected labour cost (note that 'cost' depends on productivity, which depends on extent of spillovers)

- models are 'monopolistic competitive' i.e. free entry into R&D $\Rightarrow$ zero profits (fixed cost of R&D=monopoly profits). 'Creative destruction' since new inventions destroy markets of (some) existing products.

- without 'knowledge spillovers' such firms run into diminishing returns

- such models have **three** potential market failures, which make policy implications unclear

# Market failures in R&D growth models

1. Appropriability effect (monopoly profits of a new innovation $<$ consumer surplus) $\Rightarrow$ **too little R&D**

2. Creative-destruction, or business stealing, effect (new innovation destroys profits of existing firms), which private innovator ignores $\Rightarrow$ **too much R&D**

3. Knowledge spillover effect (each firm's R&D helps reduce costs of others innovations; positive externality) $\Rightarrow$ **too little R&D**

The overall outcome depends on parameters and functional form of model

# What do we learn from such models?

- Growth of technology via 'knowledge spillovers' vital for economic growth
- Competitive profit-seeking firms can generate investment & growth, but can be market failures
- ('social planner' wants to invest more since spillovers not part of private optimisation)
- Spillovers, clusters, networks, business-university links all potentially vital
- But models too generalised to offer specific policy guidance

# Competition and growth

- Endogenous growth models imply greater competition, lower profits, lower incentive to do R&D and lower growth (R&D line shifts down)
- But this conflicts with economists' basic belief that competition is 'good'!
- Theoretical solution
  - Build models that have optimal 'competition'
  - Aghion-Howitt model describes three sector model ("escape from competition" idea)
- Intuitive idea is that 'monopolies' don't innovate

# Do ‘scale effects’ exist

- Romer model implies countries that have more ‘labour’ in knowledge-sector (e.g. R&D) should grow faster

- Jones argues this not the case (since researchers in US ↑ 5x (1950-90) but growth still ≈2% p.a.

- Hence, Jones claims his semi-endogenous model better fits the ‘facts’, BUT
  - measurement issues (formal R&D labs increasingly used)
  - ‘scale effects’ occur via knowledge externalities (these may be regional-, industry-, or network-specific)
  - Kremer (1993) suggests higher population (scale) does increase growth rates over last 1000+ years

- anyhow.... both models show $\phi$ (the ‘knowledge spillover’ parameter) is important

# Questions for discussion

1. What is the 'knife edge' property of endogenous growth models?
2. Is more competition good for economic growth?
3. Do scale effects mean that China's growth rate will always be high?