Real Business Cycle Models

Lecture 2

Nicola Viegi

April 2015

Basic RBC Model

Claim: Stochastic General Equlibrium Model Is Enough to Explain The Business cycle Behaviour of the Economy

- Money is of little importance in business cycles.
- Business cycles are created by rational agents responding optimally to real (not nominal) shocks - mostly fluctuations in productivity growth,but also fluctuations in government purchases, import prices, or preferences.

Methodology

- The economy should always be modeled using dynamic general equilibrium models.
- The quantitative implications of a proposed model should be taken seriously.

What characterizes business cycles?

- Procyclical: a variable that usually increases in booms, decreases in recessions. For example, productivity is procyclical.
- Countercyclical: a variable that usually decreases in booms, increases in recessions. For example, unemployment is countercyclical.
- Acyclical: a variable that shows no systematic relationship to the business cycle.

Business Cycle Facts

- Labor input varies considerably and procyclically
- The capital stock varies little at business cycle frequencies.
- Productivity growth is procyclical, though not nearly as much as labor input.
- Wages vary less than productivity, and have low correlation with output.
- All major expenditure categories are procyclical.
  - Investment in consumer and producer durables volatile,
  - Consumption of nondurables and services varies much less than output.
  - The most volatile expenditure category is inventory investment.

Key Puzzles

- Procyclical productivity.
  - The RBC theory explains procyclical productivity quite directly - booms are good draws of technology growth recessions are bad draws.
- Employment.
  - In the baseline RBC model, employment fluctuations are just intertemporal substitution of leisure. In some time periods, labor is less productive than others. The optimal action for workers is to work more in productive periods, less in unproductive periods. This explains unemployment.

Key Puzzles

- Persistence.
  - In the baseline RBC model, capital accumulation is the “internal propagation mechanism” - the thing that converts shocks without persistence into highly persistent shocks to output. If the technology level is below average, output is low, so investment is low, so the next period’s capital stock is also below average. So even if the technology level returns to normal next period, output will be below normal.
- Why investment spending is more variable than consumption spending.
  - This is not so hard to explain in the baseline RBC model (or any other model): an agent with the preference to smooth consumption over time will invest in productive periods and eat capital in unproductive periods.

The Model

1. The model is stochastic, so we explicitly allow for uncertainty.
2. The key source of shocks to the model are technological shocks, which are autocorrelated (typically first order autoregressive).
3. Labour supply is not fixed.

Consumer Problem

$$\max E\left[\sum_{t=0}^{\infty}\beta^{i}u\left(c_{t},1-l_{t}\right)\mid I_{0}\right] \tag{1}$$

s.t

$$k_{t+1}+c_{t}+b_{t+1}=\left(1-\delta\right)k_{t}+w_{t}l_{t}+r_{t}k_{t}+R_{t}b_{t}+\pi_{t} \tag{2}$$

Firm's problem

$$\max_{K_t, L_t} e^{z_t} F(K_t, L_t) - w_t L_t - r_t K_t$$

where F is a neoclassical production function (for example a Cobb Douglas) and z_t follows an $AR(1)$ process:

$$z_t = z_{t-1} + \varepsilon_t$$

where ε_t is white noise.
The Model is Closed with the usual Market Clearing Condition

$$y_t = c_t + i_t$$

Competitive Equilibrium

- The competitive equilibrium is unique.
- This economy satisfies the conditions that assure that both welfare theorems hold.
- Why is this important? We could solve instead the Social Planner's Problem associated with it.

Solving the Model

The consumer's Lagrangian is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} u\left(c_t, 1-l_t\right) + \\ +\lambda_t \left(\left(1-\delta\right) k_t + w_t l_t + r_t k_t + R_t b_t + \pi_t - k_{t+1} - c_t - b_{t+1} \right) \end{array} \right]$$

FOC:

$$c_t : E_t(\beta^t u_c(c_t, 1-l_t) - \lambda_t) = 0$$

$$l_t : E_t(-\beta^t u_{1-l}(c_t, 1-l_t) + \lambda_t w_t) = 0$$

$$k_{t+1} : E_t(\lambda_{t+1}(1-\delta+r_{t+1}) - \lambda_t) = 0$$

$$b_{t+1} : E_t(\lambda_{t+1}(R_t+1) - \lambda_t) = 0$$

Solving the Model

$$u_c(c_t, 1 - l_t) = \lambda_t$$

$$u_{1-l}(c_t, 1 - l_t) = u_c(c_t, 1 - l_t) w_t$$

$$u_c(c_t, 1 - l_t) = E_t \left[u_c(c_{t+1}, 1 - l_{t+1}) \left(r_{t+1} + 1 - \delta \right) \right]$$

$$R_{t+1} = E_t \left(r_{t+1} \right) + 1 - \delta$$

$$w_t = e^{z_t} F_H(K_t, H_t)$$

$$r_t = e^{z_t} F_K(K_t, H_t)$$

Calibration

- Use microeconomic studies or theory to find values for all of the parameters.
- Solve the model numerically, and simulate the economy.
- Compare the moments (standard deviations, correlations, etc) of the simulated economy with those in the actual economy.
- If the moments are matched, success!
- If not, the moments which don't match up suggest areas of potential model improvement.

Methods of Analysis

- Demonstrate that the equilibrium of the model correspond to some planner's problem, that the solution to the planner's problem corresponds to some Bellman's equation, and apply numerical dynamic programming methods.
- Transform the problem itself using a linear-quadratic approximation around the steady state. Basically, you take a quadratic approximation of the objective function and a linear approximation of the constraints. The result will be a model with linear Euler equations.
- Log-linearization of the Euler equations around the steady state. This has the advantage that it leads to closed-form approximate results, sowe can in the words of one researcher "inspect the mechanism." As with all linear approximations, this is a good approximation near the steadystate but may be a poor approximation to how the system responds to large shocks.

Simulation

- We can simulate the model on a computer and we get time series for output, employment, productivity, investment, consumption, and capital. We can look at the correlation between any of these variables, the relative variance of different variables, etc.
- There are many ways to do this simulation. The easiest is to take the Euler equations, take logs of everything, and linearize them around the balanced growth path. So the resulting variables are "detrended" - defined in terms of percentage deviation from trend. Once you have these simulated variables, you compare their properties to the properties of the corresponding detrended variables from the economy in question.