

Monetary Economics

Basic Flexible Price Models

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Modelling Money

- ▶ Cagan Model - The Price of Money
- ▶ A Modern Classical Model (Without Money)
- ▶ Money in Utility Function Approach
- ▶ Cash in Advance Models (not really)

The Cagan Model

(Cagan, P., 1956. The monetary dynamics of hyperinflation. Studies in the Quantity Theory of Money.)

- ▶ Originally a model of Hyperinflation
- ▶ Main Point: Expectation about future fundamentals determine prices now
- ▶ *Sargent, T.J. and Wallace, N., 1973. Rational expectations and the dynamics of hyperinflation. With rational expectations results are more striking*
- ▶ Used extensively in Exchange rate analysis, assets prices
- ▶ A lot of modern models look like this

The Model

Money Demand

$$\frac{M_t^d}{P_t} = L(Y_t, i_t)$$

Fisher Equation

$$(1 + i_t) = (1 + r_t) \frac{P_{t+1}}{P_t}$$

log approximation and uncertainty

$$\log(1 + i_t) = \log(1 + r_t) + E_t p_{t+1} - p_t$$

Assume Y_t and r_t fixed, the money demand equation (in log):

$$m_t^d - p_t = -\eta (E_t p_{t+1} - p_t)$$

$$m_t - p_t = -\eta (E_t p_{t+1} - p_t)$$

or

$$p_t = \frac{1}{1 + \eta} m_t + \frac{\eta}{1 + \eta} E_t p_{t+1}$$

Forward Solution

$$\begin{aligned} p_t &= \frac{1}{1+\eta} m_t + \frac{\eta}{1+\eta} E_t p_{t+1} \\ &= \frac{1}{1+\eta} m_t + \frac{\eta}{1+\eta} E_t \left[\frac{1}{1+\eta} m_{t+1} + \frac{\eta}{1+\eta} E_{t+1} p_{t+2} \right] \\ &= \frac{1}{1+\eta} \left(m_t + \frac{\eta}{1+\eta} E_t m_{t+1} \right) + \left(\frac{\eta}{1+\eta} \right)^2 E_t p_{t+2} \\ &= \dots\dots\dots \\ &= \frac{1}{1+\eta} \sum_{s=0}^{T-1} \left(\frac{\eta}{1+\eta} \right)^s m_{t+s} + \left(\frac{\eta}{1+\eta} \right)^T E_t p_{t+T} \quad (1) \end{aligned}$$

Law of Iterated Expectations

$$E_t(E_{t+1}p_{t+2}) = E_t p_{t+2}$$

No Bubble

$$\lim_{T \rightarrow \infty} \left(\frac{\eta}{1 + \eta} \right)^T E_t p_{t+T} = 0$$

$$\left| \lim_{T \rightarrow \infty} \frac{1}{1 + \eta} \sum_{s=0}^{T-1} \left(\frac{\eta}{1 + \eta} \right)^s m_{t+s} \right| < \infty$$

Solution

Generic Solution

$$p_t = \frac{1}{1 + \eta} \sum_{s=0}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^s m_{t+s} \equiv \tilde{p}_t$$

Prices at time t depend on all future expected money supply.
Expected "fundamentals" (not actual)

Constant Money Supply

$$m_t \equiv \bar{m}$$

$$E_t m_{t+s} = \bar{m} \quad (s \geq 0)$$

$$\begin{aligned} p_t &= \frac{1}{1+\eta} \sum_{s=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^s \bar{m} \\ &= \frac{1}{1+\eta} \frac{1}{1 - \frac{\eta}{1+\eta}} \bar{m} \\ &= \bar{m} \end{aligned}$$

Future One Time Money Increase at $T > 0$

Consider an surprise announcement at time $t = 0$ of a change of policy at some time in the future $T > 0$

$$m_t = \begin{array}{ll} \bar{m} & t < T \\ \bar{m}' > \bar{m} & t \geq T \end{array}$$

Thus

$$p_t = \begin{array}{ll} \bar{m} & t < T \\ \bar{m}' & t \geq T \end{array}$$

How would the transition look like?

Solution

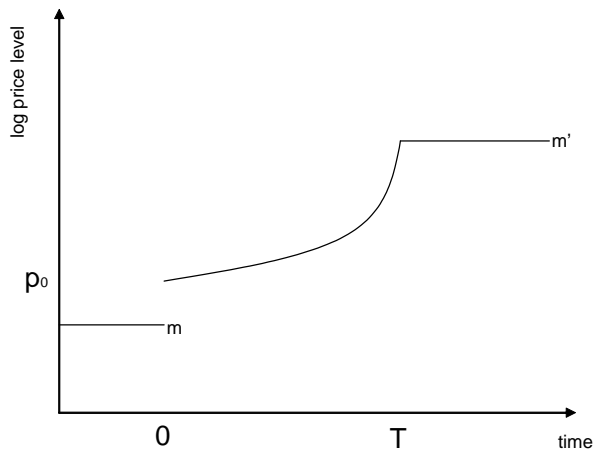
In the period between $0 \leq t \leq T$

$$p_t = \frac{1}{1+\eta} \sum_{s=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^s \bar{m} + \frac{1}{1+\eta} \sum_{s=T-t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^s (\bar{m}' - \bar{m})$$

$$p_t = \bar{m} + \frac{1}{1+\eta} \left(\frac{\eta}{1+\eta} \right)^{T-t} \sum_{s=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^s (\bar{m}' - \bar{m})$$

$$p_t = \bar{m} + \left(\frac{\eta}{1+\eta} \right)^{T-t} (\bar{m}' - \bar{m})$$

Solution



- ▶ Prices "Jump" at the announcement, before the policy is implemented
- ▶ Expectations drive economic dynamics

A Classical Monetary Model (Gali Ch. 1)

- ▶ Why Classical?
 - ▶ Perfect Competition
 - ▶ Fully Flexible Prices
- ▶ Real Business Cycle Structure + Nominal Prices Determination
- ▶ Three Agents: Households, Firms, Central Bank (setting nominal interest rate)

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for $t = 0, 1, 2, \dots$ plus solvency constraints

Optimality Conditions

$$U_{n,t} + \frac{W_t}{P_t} U_{c,t} = 0$$

$$\frac{Q_t}{P_t} U_{c,t} = \beta E_t \left\{ U_{c,t+1} \frac{1}{P_{t+1}} \right\}$$

Households

Specification of Utility

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

which gives the log-linear optimality conditions (aggregate variables)

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - \rho)$$

where $i_t = \log Q_t$ is the nominal interest rate, $\rho = -\log \beta$ is the discount rate, and $\pi_{t+1} = p_{t+1} - p_t$ is the inflation rate

Firms

Profit Maximization

$$\max P_t Y_t - W_t N_t$$

subject to

$$Y_t = A_t N_t^{1-\alpha}$$

Optimality Conditions

$$\frac{W_t}{P_t} = (1 - \alpha) A_t N_t^{-\alpha}$$

log linear version (ignoring constants)

$$w_t - p_t = a_t - \alpha n_t$$

Equilibrium

Equilibrium Value of Real Variables

$$y_t = c_t$$

Labour Market

$$\sigma c_t + \varphi n_t = a_t - \alpha n_t$$

Asset market Clearing

$$b_t = 0$$

$$y_t = E_t \{y_{t+1}\} - \frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - \rho)$$

Aggregate Production relationship

$$y_t = a_t + (1 - \alpha) n_t$$

Equilibrium

Equilibrium Values of Real Variables

$$n_t = \frac{1 - \sigma}{\sigma(1 - \alpha) + \varphi + \alpha} a_t; y_t = \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} a_t$$

$$w_t - p_t = \frac{\sigma + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} a_t$$

$$r_t \equiv i_t - E_t \{ \pi_{t+1} \} = \rho - \frac{\sigma(1 + \varphi)(1 - \rho_a)}{\sigma(1 - \alpha) + \varphi + \alpha} a_t$$

- ▶ Real variables determined independently of monetary policy (neutrality)
- ▶ Optimal Policy Undetermined
- ▶ Specification of Monetary Policy Needed to determine nominal variables

Monetary Policy and Price Level Determination

A Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \pi_t$$

combined with the definition of real rate $r_t \equiv i_t - E_t \{\pi_{t+1}\}$,

$$\phi_\pi \pi_t \equiv E_t \{\pi_{t+1}\} + \widehat{r}_t$$

if $\phi_\pi > 1$, unique stationary solution

$$\pi_t \equiv \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} \{\widehat{r}_{t+k}\}$$

if $\phi_\pi < 1$, any process π_t satisfying

$$\pi_{t+1} \equiv \phi_\pi \pi_t - \widehat{r}_t + \xi_{t+1}$$

where $E_t \{\xi_{t+1}\} = 0$ for all t is consistent with a stationary equilibrium (Price Level Indeterminacy)

- ▶ Illustration of the "Taylor Principle"

Monetary Policy and Price Level Determination

Exogenous Path of Money Supply (Cagan Again)

$$m_t - p_t = y_t - \eta i_t$$

Combining Money Demand and Fisher Equation

$$r_t \equiv i_t - E_t \{ \pi_{t+1} \},$$

$$p_t = \left(\frac{\eta}{1 + \eta} \right) E_t \{ p_{t+1} \} + \left(\frac{1}{1 + \eta} \right) m_t + u_t$$

where $u_t = \left(\frac{1}{1 + \eta} \right) (\eta r_t - y_t)$ evolves independently of m .

Assuming $\eta > 1$ and solving forward

$$p_t = \frac{1}{1 + \eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^k E_t \{ m_{t+k} \} + u'_t$$

where $u'_t = \sum_{k=0}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^k E_t \{ u_{t+k} \}$

Monetary Policy and Price Level Determination

Exogenous Path of Money Supply (Cagan Again)

Rewrite in term of expected future money growth rates

$$p_t = m_t + \frac{1}{1 + \eta} \sum_{k=1}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^k E_t \{ \Delta m_{t+k} \} + u'_t$$

which implies the following nominal interest rate:

$$\begin{aligned} i_t &= \eta^{-1} [y_t - (m_t - p_t)] \\ i_t &= \eta^{-1} \frac{1}{1 + \eta} \sum_{k=1}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^k E_t \{ \Delta m_{t+k} \} + v_t \end{aligned}$$

where $v_t \equiv \eta^{-1} [y_t + u'_t]$ is independent of policy

Money in The Utility Function

Money in the Utility function is just a modelling trick to determine money demand inside the private sector optimization

The household's optimization problem (with fixed labour supply)

$$\max_{\{C_t, M_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u \left(C_t, \frac{M_t}{P_t} \right)$$

subject to the real budget constraint

$$K_{t+1} + \frac{M_t}{P_t} = (1 + r_t) K_t + \frac{M_{t-1}}{P_t} + w_t - C_t - T_t$$

Production is given by a neoclassical production function with constant return to scale

$$Y_t = F(K_t, L_t)$$

Equilibrium Conditions

$$u_c \left(C_t, \frac{M_t}{P_t} \right) = \beta (1 + r_{t+1}) u_c \left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}} \right) \quad (2)$$

$$u_c \left(C_t, \frac{M_t}{P_t} \right) = u_{M/P} \left(C_t, \frac{M_t}{P_t} \right) + \beta u_c \left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}} \right) \frac{P_t}{P_{t+1}} \quad (3)$$

Substituting for $\beta u_c \left(C_{t+1}, M_{t+1}/P_{t+1} \right)$ from (2) in (3) and rearrange, we get:

$$u_c \left(C_t, \frac{M_t}{P_t} \right) \left(1 - \frac{1}{(1 + r_{t+1})} \frac{P_t}{P_{t+1}} \right) = u_{M/P} \left(C_t, \frac{M_t}{P_t} \right) \quad (4)$$

Equilibrium Conditions

Remember the Fisher Equation that relates real and nominal interest rates

$$(1 + i_t) = E_t (1 + r_{t+1}) \frac{P_{t+1}}{P_t}$$

under perfect foresight, (4) can then be written as:

$$\frac{i_t}{1 + i_t} = u_{M/P} \left(C_t, \frac{M_t}{P_t} \right) / u_c \left(C_t, \frac{M_t}{P_t} \right) \quad (5)$$

LM Equation

General Equilibrium in MIU

Need to Specify:
Government Behaviour

$$-T_s = \frac{M_s - M_{s-1}}{P_s}$$

Factor prices

$$r_t = \frac{\partial F(K, 1)}{\partial K}$$
$$w_t = F(K, 1) - K_t \frac{\partial F(K, 1)}{\partial K}$$

where the second relation uses the property of constant return to scale ($F = L\partial F/\partial L + K\partial F/\partial K$)

Steady State

To be consistent with the data a model including money should have the following

- ▶ Neutrality of Money: the real equilibrium is independent of the money stock
- ▶ Superneutrality of money: the real equilibrium is independent of the money growth rate

Nominal Equilibrium

$$M_t / M_{t-1} = 1 + \sigma$$

$$P_t / P_{t-1} = 1 + \pi$$

$$C_t / C_{t-1} = 1$$

$$M_t / P_t / M_{t-1} / P_{t-1} = 1$$

$$\pi = \sigma$$

Real Equilibrium

from

$$u_c \left(C_t, \frac{M_t}{P_t} \right) = \beta (1 + r_{t+1}) u_c \left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}} \right)$$

In steady state

$$u_c \left(C, \frac{M}{P} \right) = \beta (1 + r_{t+1}) u_c \left(C, \frac{M}{P} \right)$$

$$(1 + r) = 1/\beta$$

Combining with $r_t = \frac{\partial F(K,1)}{\partial K}$, gives that the steady state capital stock must solve

$$F_K (K, 1) = r = 1/\beta - 1$$

which depends only on the technology and the real discount rate, not on the money stock or growth. Because capital stock is constant, so consumption in steady state is uniquely determined by the real side of the economy

Monetary Equilibrium

In steady state, in this model, money is neutral and superneutral
With a value for the steady state consumption, we can solve for a value of the steady state real money balances by combining the Fisher equation

$$(1 + i) = E_t (1 + r) P_{t+1} / P_t$$

$$(1 + i) = E_t (1 + r) (1 + \pi)$$

and

$$\frac{i}{1+i} = u_{M/P} \left(C, \frac{M}{P} \right) / u_c \left(C, \frac{M}{P} \right)$$

would give an equation in steady state real money and of known parameters

Optimal Rate of Inflation

Friedman (1969)

Social Marginal cost of producing money = 0

Private marginal cost of money = i

$$PMC = SMC$$

$$i = 0$$

$$(1 + i) = E_t (1 + r) \frac{P_{t+1}}{P_t}$$

$$i = r + \pi = 0$$

$$\pi = -r$$

Deflation

Cost Of Inflation

Money Demand Function

$$\frac{i}{1+i} = u_{M/P} \left(C, \frac{M}{P} \right) / u_C \left(C, \frac{M}{P} \right)$$

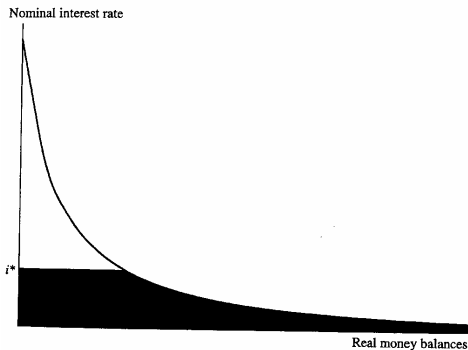


Figure 2.2
The Welfare Costs of Inflation

Conclusions

- ▶ Basic Flexible Price Models Highlight:
 - ▶ The Role of Expectations
 - ▶ The Importance of Policy Design for Price Determination
 - ▶ Cost of Inflation (even when we have neutrality of money)
- ▶ Problems:
 - ▶ Money has no effect on real variables (only welfare)
 - ▶ Friedman (1968) not realistic (Deflation is costly, but why?)