Macroeconomics

Basic New Keynesian Model

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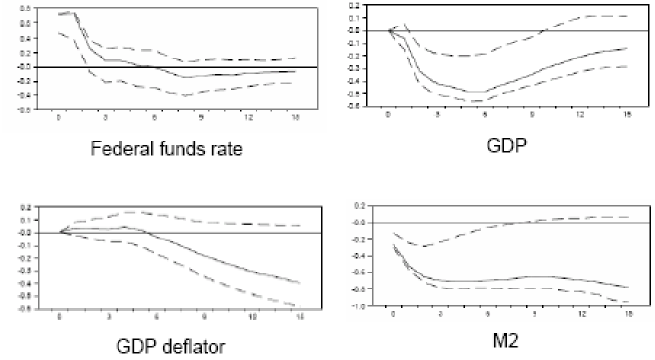
The Problem

- Short run Effects of Monetary Policy Shocks
  - persistent effects on real variables
  - slow adjustment of aggregate price level
  - liquidity effect
- Micro Evidence on Price-setting Behaviour: significant price and wage rigidities
- Failure of Classical Monetary Models

The Evidence

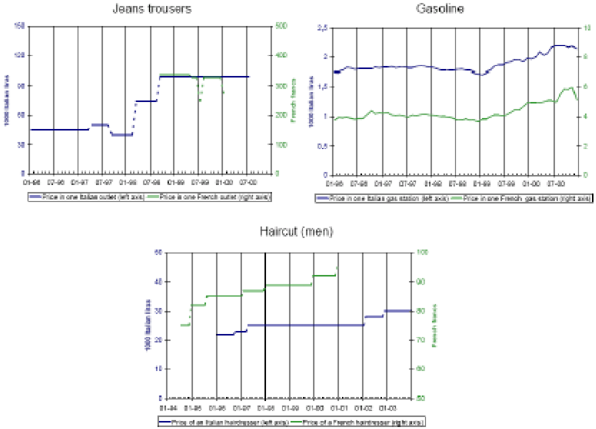
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

The Evidence

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)

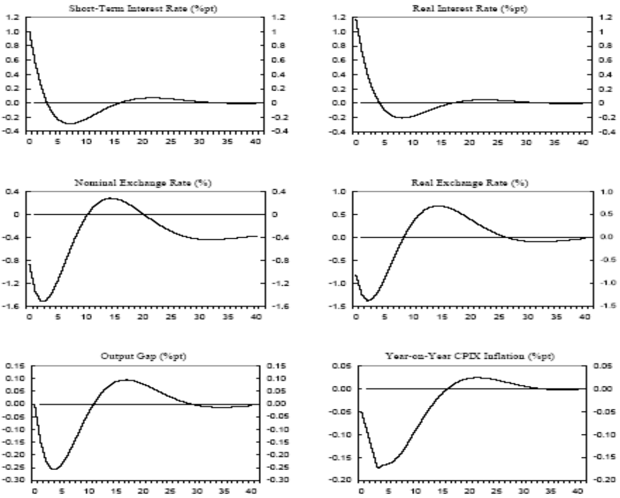


Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al.* (2004) and Veronese *et al.* (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. WP 05

The Evidence: South Africa

Figure 2. Interest Rate Shock
(deviations from control)



Why New Keynesian?

- Real Rigidities - Monopolistic Competition
- Nominal Rigidities - Calvo Pricing
- But with Microfundations
  - Method RBC literature (Dynamic Stochastic General Equlibrium)
  - Explain Persistence and Demand Shocks

The Basic Macro Structure

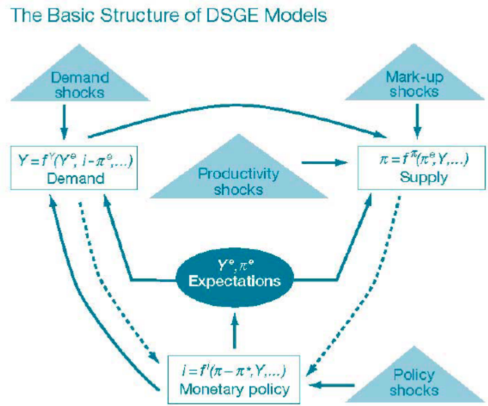


Figure 1. The basic structure of DSGE models. Source: Sbordone *et al.* (2010).

The Simple Macro Structure

Supply (NK Phillips Curve)

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \tag{1}$$

Demand (NK IS)

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + \eta_t \tag{2}$$

Monetary Policy Rule (Taylor Rule)

$$i_t = \bar{r}_t + \phi_\pi \left(\pi_t - \pi^* \right) + \phi_y \left(y_t \right) \tag{3}$$

Where Do They Come From?

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \tag{4}$$

Derived from optimal pricing behaviour of the firm with market power and nominal stickiness (Calvo Pricing)

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + \eta_t \tag{5}$$

Directly from the first order condition of the consumer and equilibrium conditions

$$i_t = \bar{r}_t + \phi_\pi \left(\pi_t - \pi^* \right) + \phi_y \left(y_t \right) \tag{6}$$

Monetary policy reaction function
Note:

- There is no need of having an LM (no explicit money)
- Both π_t and y_t are jumping variables (function of expectations of future state variables)

Consumer Problem

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right] \tag{7}$$

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{8}$$

$$P_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \tag{9}$$

$$P_t C_t + B_t = W_t N_t + (1 + r_t) B_{t-1} + \Pi_t \tag{10}$$

Consumer Problem

Optimal Allocation of Consumption Expenditure

$$\min_{c_{ij}} \int_0^1 p_{jt} c_{jt} dj \tag{11}$$

subject to

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{12}$$

Optimal Allocation

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t \tag{13}$$

Consumer Problem

Optimal Dynamic Consumption/Leisure Decision

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right] \tag{14}$$

$$C_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + (1 + r_t) \frac{B_{t-1}}{P_t} + \frac{\Pi_t}{P_t} \tag{15}$$

Lagrangian

$$L = E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right] + \tag{16}$$

$$+ E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \left[\begin{array}{c} \frac{W_{t+i}}{P_{t+i}} N_{t+i} + (1 + r_{t+i}) \frac{B_{t+i-1}}{P_{t+i}} \\ + \frac{\Pi_{t+i}}{P_{t+i}} - C_{t+i} - \frac{B_{t+i}}{P_{t+i}} \end{array} \right] \tag{17}$$

Consumer Problem

Optimal Dynamic Consumption/Leisure Decision

FOC

$$\frac{\partial L}{\partial C_t} = C_t^{-\sigma} + \lambda_{t+i} = 0 \quad (18)$$

$$\frac{\partial L}{\partial N_t} = -\chi N_t^{\eta} + \lambda_t \frac{W_t}{P_t} = 0 \quad (19)$$

$$\frac{\partial L}{\partial B_t} = -\lambda_t \frac{1}{P_t} + E_t\left[\beta \lambda_{t+1} \left(1 + r_{t+1}\right) \frac{1}{P_{t+i}}\right] = 0 \quad (20)$$

Consumer Problem

Optimal Dynamic Consumption/Leisure Decision

Rearranging and using condition (18) to eliminate the Lagrange multiplier, we get:

$$\chi N_t^{\eta} = C_t^{-\sigma} \frac{W_t}{P_t} \tag{21}$$

$$C_t^{-\sigma} = E_t \left[\beta \left(1 + r_{t+1}\right) \left(\frac{P_t}{P_{t+i}} \right) C_{t+1}^{-\sigma} \right] \tag{22}$$

As shown before, this implies the following log linear relationships

$$w_t - p_t = \sigma c_t + \eta n_t \tag{23}$$

$$c_t = E_t \left\{ c_{t+1} \right\} - \frac{1}{\sigma} \left\{ i_t - E_t \pi_{t=1} - \ln \beta \right\} \tag{24}$$

Firm Problem

Calvo Pricing

- A firm may change price of its product only when Calvo Fairy visits.
- The probability of a visit is $(1 - \theta)$
- It is independent of the length of the time and the time elapsed since the last adjustment. Hence, in each period the $(1 - \theta)$ share of firms
- may change their price and rest, θ, keep their price unchanged.
- Mathematically, Calvo Fairy's visits follows Bernoulli process (discrete version of Poisson process).
- The probability distribution of the number of periods between the visits of Calvo Fairy is geometric distribution.
- The expected value of geometric distribution and, hence, the average number of periods between the price changes (of a firm) is

$$\frac{1}{1 - \theta}$$

Firm Problem

Calvo Pricing

Price Stickiness: Firms can change prices with a probability $(1-\theta)$
Price Decision - Intertemporal
Problem of the Firm (when they can change the prices)

$$\min_{z_t} L(z_t) = \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left(z_t - p^*_{t+k}\right)^2 \tag{25}$$

FOC

$$L'(z_t) = 2\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(z_t - p^*_{t+k}\right) = 0 \qquad (26)$$

$$\left[\sum_{k=0}^{\infty}(\theta\beta)^k\right] z_t = \sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(p^*_{t+k}\right) \qquad (27)$$

$$\frac{z_t}{1-\theta\beta} = \sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(p^*_{t+k}\right) \qquad (28)$$

$$z_t = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(p^*_{t+k}\right) \qquad (29)$$

Reset Price

$$z_t = (1 - \theta\beta) p_t^* + \theta\beta E_t z_{t+1} \tag{30}$$

where

$$E_t z_{t+1} = \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left(p_{t+1+k}^*\right)$$

"frictionless optimal" price p^*.

$$p_t^* = \mu + mc_t \tag{31}$$

Thus the reset price can be written as:

$$z_t = (1 - \theta\beta)(\mu + mc_{t+k}) + (1 - \theta\beta) E_t z_{t+1} \tag{32}$$

Aggregate Pricing

$$p_t = \theta p_{t-1} + (1-\theta)\, z_t \tag{33}$$

This can be rearranged as

$$z_t = \frac{1}{1-\theta}\left(p_t - \theta p_{t-1}\right) \tag{34}$$

$$\frac{1}{1-\theta}\left(p_t - \theta p_{t-1}\right) = \frac{\theta\beta}{1-\theta}\left(p_{t+1} - \theta p_t\right) + (1-\theta\beta)\left(\mu + mc_t\right)$$

this equation implies:

$$\pi_t = \beta\pi_{t+1} + \frac{1-\theta}{\theta}(1-\theta\beta)\left(\mu + mc_t - p_t\right) \tag{35}$$

New Keynesian Phillips Curve

Denoting $\widehat{mc}_t = \mu + mc_t - p_t$

$$\pi_t = \beta \pi_{t+1} + \frac{1-\theta}{\theta} (1 - \theta\beta) \widehat{mc}_t \tag{36}$$

$$\widehat{mc}_t = \lambda y_t$$

$$\pi_t = \beta \pi_{t+1} + \gamma y_t \tag{37}$$

where

$$\gamma = \frac{\lambda (1-\theta)(1-\theta\beta)}{\theta}$$

Inflation as an Asset Price

Solving Forward the New Keynesian Phillips curve (as for the Cagan Model)

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t y_{t+k}$$

Problems

- Not enough dynamic in the model
- Any change will be reflected instantaneously on the variables
- Many attempts to produce more slugghish response (will see later)

System Stability

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \quad (38)$$

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + \eta_t \quad (39)$$

:

$$i_t = \bar{i}_t + \phi_\pi \left(\pi_t - \pi^* \right) + \phi_y \left(y_t \right) \quad (40)$$

Where ϕ_π, $\phi_y \geq 0$.

Substituting the general (40) now in the above system can be written in state-space form:

$$E_t \mathbf{z}_{t+1} = \widetilde{\mathbf{A}} * \mathbf{z}_t + \widetilde{\mathbf{B}} * \pi^* + \mathbf{K}\mathbf{u}_{t+1}$$

where, $\mathbf{z}'_t = \begin{bmatrix} \pi_t & y_t \end{bmatrix}$, $\mathbf{u}'_t = \begin{bmatrix} \varepsilon_t & \eta_t & \bar{i}_t \end{bmatrix}$ and

$$\widetilde{\mathbf{A}} = \begin{pmatrix} \frac{1}{\beta} & -\frac{\alpha}{\beta} \\ -\frac{\gamma}{\beta} + \gamma\phi_\pi & \frac{\alpha\gamma}{\beta} + 1 + \gamma\phi_y \end{pmatrix}$$

Stability of the system eigenvalues of matrix **A** outside the unit circle (Blanchard and Kahn, 1980).

This implies the following conditions

$$\left|\frac{tr(\widetilde{\mathbf{A}}) \pm \sqrt{tr(\widetilde{\mathbf{A}})^2 - 4\det(\widetilde{\mathbf{A}})}}{2}\right| > 1$$

which, given our parameter restrictions and the nature of the problem, reduces to the following three conditions (See Woodford 2003 for detailed derivation)

$$\det(\widetilde{\mathbf{A}}) > 0, \qquad \det(\widetilde{\mathbf{A}}) - tr(\widetilde{\mathbf{A}}) > -1, \qquad \det(\widetilde{\mathbf{A}}) + tr(\widetilde{\mathbf{A}}) > -1$$

Necessary Condition For Stability

$$\left(\frac{1-\beta}{\alpha}\right)\phi_y + \phi_\pi > 1 \tag{41}$$

Sufficient Condition

$$\phi_\pi > 1 \tag{42}$$

Taylor Rule: for the system to be stable the CB should react with an elasticity of interest rate relative to inflation greater than one (Taylor proposed $\phi_\pi = 1.5$ and $\phi_y = 0.5$)