Monetary Economics

Optimal Monetary Policy in New Keynesian Models

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Monetary Policy in New Keynesian Models

- Why Monetary Policy?
- For What?
- How?

Monetary Policy in New Keynesian Models

- Two Distortions:
  - Monopolistic Competition
    - (Mark up over marginal cost): to be corrected with taxes (optimal subsidy)
  - Sticky Prices
    - Average Mark Up veries over time in response to shocks
    - Relative prices changes because of staggered price adjustment
- Monetary Policy to adjust the Distortion Caused by Sticky Prices

Monetary Policy Objectives

- Traditional Economic Policy Theory - Quadratic Loss Function (ad hoc, no microfundations)

$$L = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left[(\pi_{t+i})^2 + \theta \left(y_{t+i} - \overline{y} \right)^2 \right]$$

- The structure of the New Keynesian Model gives a rationale for the quadratic objective function - (Woodford Benigno 2005, Gali Appendix Chapter 4)
  - Fluctuations of prices produces deviation of relative prices
  - Fluctuations of Output produces fluctuation of mark up
  - Both Fluctuations affect the Utility of Individuals - Utility Based Objective Function

$$L = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left[\left(\frac{\varepsilon}{\lambda} \right) (\pi_{t+i})^2 + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) (y_{t+i})^2 \right]$$

Optimal Monetary Policy in New Keynesian Models

Three approaches:

- Plug an ad hoc interest rate rule in the model (Taylor Rule) and study the dynamic characteristics (stability, learnability etc) and models response to shocks. Search for the "best simple rule"
- Define a specific objective function for Central Bank and Optimize (as approximation of social objective function - Benigno and Woodford,2005)
- A Combination of the two (optimize in a subset of possible simple rules)

Taylor Rule

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \quad (1)$$

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + \eta_t \quad (2)$$

:

$$i_t = \bar{i}_t + \phi_\pi \left(\pi_t - \pi^* \right) + \phi_y \left(y_t \right) \quad (3)$$

Where ϕ_π, $\phi_y \geq 0$.

State Space Representation

Substituting the general (3) now in the above system can be written in state-space form:

$$E_t \mathbf{z}_{t+1} = \widetilde{\mathbf{A}} * \mathbf{z}_t + \widetilde{\mathbf{B}} * \pi^* + \mathbf{K}\mathbf{u}_{t+1}$$

where, $\mathbf{z}'_t = \begin{bmatrix} \pi_t & y_t \end{bmatrix}$, $\mathbf{u}'_t = \begin{bmatrix} \varepsilon_t & \eta_t & \bar{i}_t \end{bmatrix}$ and

$$\widetilde{\mathbf{A}} = \begin{pmatrix} \frac{1}{\beta} & -\frac{\alpha}{\beta} \\ -\frac{\gamma}{\beta} + \gamma\phi_\pi & \frac{\alpha\gamma}{\beta} + 1 + \gamma\phi_y \end{pmatrix}$$

Stability of the system eigenvalues of matrix **A** outside the unit circle (Blanchard and Kahn, 1980).

Stability Condition

This implies the following conditions

$$\left| \frac{tr(\widetilde{\mathbf{A}}) \pm \sqrt{tr(\widetilde{\mathbf{A}})^2 - 4\det(\widetilde{\mathbf{A}})}}{2} \right| > 1$$

which, given our parameter restrictions and the nature of the problem, reduces to the following three conditions (See Woodford 2003 for detailed derivation)

$$\det(\widetilde{\mathbf{A}}) > 0, \qquad \det(\widetilde{\mathbf{A}}) - tr(\widetilde{\mathbf{A}}) > -1, \qquad \det(\widetilde{\mathbf{A}}) + tr(\widetilde{\mathbf{A}}) > -1$$

Stability Condition

Necessary Condition For Stability

$$\left(\frac{1-\beta}{\alpha}\right)\phi_y + \phi_\pi > 1 \tag{4}$$

Sufficient Condition

$$\phi_\pi > 1 \tag{5}$$

Taylor Rule: for the system to be stable the CB should react with an elasticity of interest rate relative to inflation greater than one (Taylor proposed $\phi_\pi = 1.5$ and $\phi_y = 0.5$)

Optimal Monetary Policy - Discretion Vs Commitment

- Discretion vs Commitment in Traditional Models of Monetary Policy - Kydland and Prescott (1977), Barro Gordon (1983), Rogoff (1985)
  - Discretionary Policy Inefficient Because CB Objectives are Time Inconsistent
- Discretion vs Commitment in New Keynesian Models of Monetary Policy, Clarida, Gali Gertler (1999), Woodford (2003).
  - Discretionary Policy Inefficient Because CB does not internalise dynamic properties of the model
- History Dependent vs Forward Looking Monetary Policy
  - The nature of optimal policy depends on the nature of private sector expectations

Optimal Policy Problem

$$\min_{y} L_t = E_t \sum_{\tau=0}^{\infty} \frac{1}{2}\beta^{\tau} \left\{ \left(\pi_{t+\tau} - \pi^T \right)^2 + \gamma y_{t+i}^2 \right\} \tag{6}$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + k y_t + \varepsilon_t \tag{7}$$

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + \xi_t \tag{8}$$

Lagrangian

$$\min_i L_t =$$

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left\{ \begin{array}{c} \left[\frac{1}{2}\left(\pi_{t+\tau}^2 + \gamma y_{t+i}^2\right)\right] + \\ \lambda_{t+\tau}\left[\pi_{t+\tau} - \beta E_t \pi_{t+1+\tau} - k y_{t+\tau} - \varepsilon_{t+\tau}\right] \\ +\mu_{t+\tau}\left[y_{t+\tau} - E_t y_{t+1+\tau} + \gamma\left(i_{t+\tau} - E_t \pi_{t+1+\tau}\right) - \xi_{t+\tau}\right] \end{array} \right\}$$

FOC respect to $i_{t+\tau}$ is equal to

$$\mu_{t+\tau}\gamma = 0$$

- Problem can be stated just in term of π and y

Discretionary Solution

Period by period problem

$$\min_{y} L = \frac{1}{2} E \left[\pi_t^2 + \gamma y_t^2 \right] \tag{9}$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + k y_t + \varepsilon_t \tag{10}$$

$$\varepsilon_{t+1} = \rho \varepsilon_t + v_t, \qquad 0 < \rho < 1, \qquad v_t \text{ is } iid$$

FOC

$$\frac{dL}{dy_t} = k \pi_t + \gamma y_t = 0 \tag{11}$$

$$y_t = -\frac{k}{\gamma} \pi_t \tag{12}$$

Solution

$$\pi_t = \frac{\gamma}{\gamma + k^2} \beta E_t \pi_{t+1} + \frac{\gamma}{\gamma + k^2} \varepsilon_t. \tag{13}$$

Solving forward

$$\pi_t = \frac{\gamma}{k^2 + \gamma (1 - \beta\rho)} \varepsilon_t \tag{14}$$

Commitment - Timeless Perspective

$$\min_{i} L_t = E_t \sum_{\tau=0}^{\infty} \frac{1}{2} \beta^{\tau} \left(\pi_{t+\tau}^2 + \gamma y_{t+i}^2 \right) \tag{15}$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + k y_t + \varepsilon_t$$

where, as before, $\varepsilon_{t+1} = \rho \varepsilon_t + v_t$ is the stochastic policy process

FOC

$$\min_i L_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left\{ \begin{array}{c} \left[\frac{1}{2}\left(\pi_{t+\tau}^2 + \gamma y_{t+i}^2\right)\right] + \\ +\lambda_{t+\tau}\left[\pi_{t+\tau} - \beta E_t \pi_{t+1+\tau} - k y_{t+\tau} - \varepsilon_{t+\tau}\right] \end{array} \right\} \tag{16}$$

FOC with respect to π_t and y_t for $\tau = 0$

$$\frac{\delta L}{\delta \pi_t} = \pi_t + \lambda_t = 0 \tag{17}$$

$$\frac{\delta L}{\delta y_t} = \gamma y_t - k\lambda_t = 0 \tag{18}$$

and for $\tau > 0$

$$\frac{\delta L}{\delta \pi_{t+\tau}} = \beta^{\tau}\left(\pi_{t+\tau} + \lambda_{t+\tau} - \lambda_{t+\tau-1}\right) = 0 \tag{19}$$

$$\frac{\delta L}{\delta y_{t+\tau}} = \gamma y_{t+\tau} - k\lambda_t = 0 \tag{20}$$

FOC (2)

$$
\begin{aligned}
for.\tau &\geq 0 \rightarrow \frac{\delta L}{\delta y_{t+\tau}} = \gamma y_{t+\tau} - k\lambda_t = 0 &(21)\\
for.\tau &= 0 \rightarrow \frac{\delta L}{\delta \pi_t} = \pi_t + \lambda_t = 0 &(22)\\
for.\tau &> 0 \rightarrow \frac{\delta L}{\delta \pi_{t+\tau}} = \pi_{t+\tau} + \lambda_{t+\tau} - \lambda_{t+\tau-1} = 0 &(23)
\end{aligned}
$$

Policy Inertia

Combining (23) and (21) we get

$$\pi_t + \frac{\gamma}{k}(y_t - y_{t-1}) = 0 \tag{24}$$

which can be rewritten as

$$y_t = y_{t-1} - \frac{k}{\gamma}\pi_t \tag{25}$$

Inertia in the policy rule?

Solution

substituting (24) in the supply equation

$$
\begin{aligned}
-\frac{\gamma}{k}\left(y_t - y_{t-1}\right) &= -\beta\frac{\gamma}{k}\left(E_t y_{t+1} - y_t\right) + k y_t + \varepsilon_t \\
-\frac{\gamma}{k} y_t - \beta\frac{\gamma}{k} y_t - k y_t &= -\beta\frac{\gamma}{k} E_t y_{t+1} - \frac{\gamma}{k} y_{t-1} + \varepsilon_t \\
\left(1 + \beta + \frac{k^2}{\gamma}\right) y_t &= \beta E_t y_{t+1} + y_{t-1} - \frac{k}{\gamma}\varepsilon_t \qquad (26)
\end{aligned}
$$

Second order difference equation to solve with **undetermined coefficients method**

Undetermined Coefficients Method

First posit a solution for y_t that is a function of the state $(y_{t-1,}\varepsilon_t)$:

$$y_t = ay_{t-1} + b\varepsilon_t$$

This, togheter with the assumption of shocks following a AR(1) process, gives

$$E_t y_{t+1} = ay_t + b\rho\varepsilon_t = a^2 y_{t-1} + b\left(a + \rho\right)\varepsilon_t$$

Undetermined Coefficients Method

substituting in (26) we get

$$\left(1+\beta+\frac{k^2}{\gamma}\right)\left(ay_{t-1}+b\varepsilon_t\right)=\beta\left[a^2y_{t-1}+b\left(a+\rho\right)\varepsilon_t\right]+y_{t-1}-\frac{k}{\gamma}\varepsilon_t$$

that is:

$$\left(1+\beta+\frac{k^2}{\gamma}\right)\left(ay_{t-1}+b\varepsilon_t\right)=\left(1-\beta a^2\right)y_{t-1}+\left(\beta b\left(a+\rho\right)-\frac{k}{\gamma}\right)\varepsilon_t$$

Undetermined Coefficients Method

Find the value of a and b that match the coefficients:

$$\left(1+\beta+\frac{k^2}{\gamma}\right)a=\left(1-\beta a^2\right)_t$$

$$\left(1+\beta+\frac{k^2}{\gamma}\right)b=\beta b\left(a+\rho\right)-\frac{k}{\gamma}$$

Equation for a quadratic - choose the root $|a|<1$. The solution for b is instead unique, i.e

$$b=-\left(\frac{k}{\gamma\left(1+\beta\left(1-a-\rho\right)\right)+k^2}\right)$$

Comparison

Decision rule for π_t is:

$$\begin{aligned}
\pi_t &= -\frac{\gamma}{k}\left(y_t - y_{t-1}\right) \\
\pi_t &= -\frac{\gamma}{k}\left(ay_{t-1} - \left(\frac{k}{\gamma\left(1+\beta\left(1-a-\rho\right)\right)+k^2}\right)\varepsilon_t - y_{t-1}\right) \\
\pi_t &= \frac{\gamma}{k}\left(1-a\right)y_{t-1} + \left(\frac{\gamma}{\gamma\left(1+\beta\left(1-a-\rho\right)\right)+k^2}\right)\varepsilon_t
\end{aligned}$$

Compare with Solution under discretion:

$$\pi_t = \frac{\gamma}{k^2+\gamma\left(1-\beta\rho\right)}\varepsilon_t \tag{27}$$

Conclusions

- Under both precommitment and discretion, monetary policy completely offsets the impacts of the demand shock, so ξ_t does not affect either y or π.
- Cost shocks, ε_t, have different impacts under precommitment and discretion because under discretion there is a stabilization bias.
- There is history dependence of optimal policy under precommitment, but none under discretion. The history dependence is the way commitment is implemented.