Stability of New Keynesian Models

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UP

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right) + \eta_t \tag{1}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \varepsilon_t \tag{2}$$

Interest rate rule

$$i_t = r_t^n \tag{3}$$

Substitute the rule in the structural equation

$$y_t = E_t y_{t+1} + \frac{1}{\sigma}\left(E_t \pi_{t+1}\right) + \eta_t \tag{4}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \varepsilon_t \tag{5}$$

Write the system in the form $X_{t+1} = AX_t$

$$E_t y_{t+1} + \frac{1}{\sigma}\left(E_t \pi_{t+1}\right) = y_t - \eta_t \tag{6}$$
$$\beta E_t \pi_{t+1} = \pi_t - \kappa y_t + \varepsilon_t \tag{7}$$

$$\begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \eta_t \\ \epsilon_t \end{bmatrix}$$

that is equal to

$$\begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \eta_t \\ \epsilon_t \end{bmatrix}$$

For the stability of the system you need to study the stability of the characteristic matrix

$$A = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix}$$

Inverse of $\begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix}^{-1} = \frac{1}{\beta} \begin{bmatrix} \beta & -\frac{1}{\sigma} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{\sigma\beta} \\ 0 & \frac{1}{\beta} \end{bmatrix}$

thus

$$A = \begin{bmatrix} 1 & -\frac{1}{\sigma\beta} \\ 0 & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{k}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ -\frac{k}{\beta} & \frac{1}{\beta} \end{bmatrix}$$

With two jumping variables we expect two eigenvalues greater that 1 for stability (notice that the book does the inverse problem). Use the computer to find the solution

$$\begin{aligned} \kappa &= 0.024 \\ \sigma &= 6.25 \\ \beta &= 0.99 \end{aligned}$$

Structure of the octave file

```
k=0.024
s=6.25
b=0.99
A=[1+(k/(s*b)),-1/(s*b); -k/b, 1/b]
eig(A)
```

0.1 Exercise

Analyse the stability of the following system (using the previous parameter values and experimenting with ϕ_π and ϕ_y)

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - r_t^n \right) + \eta_t \tag{8}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \varepsilon_t \tag{9}$$

$$i_t = \bar{i}_t + \phi_\pi \left(\pi_t - \pi^* \right) + \phi_y \left(y_t \right) \tag{10}$$