

Macroeconomics

Basic New Keynesian Model

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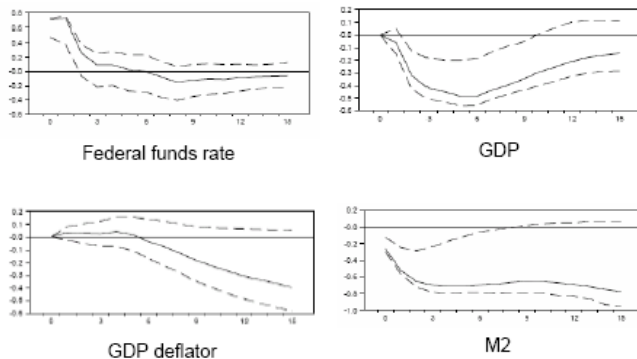
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The Problem

- ▶ Short run Effects of Monetary Policy Shocks
 - ▶ persistent effects on real variables
 - ▶ slow adjustment of aggregate price level
 - ▶ liquidity effect
- ▶ Micro Evidence on Price-setting Behaviour: significant price and wage rigidities
- ▶ Failure of Classical Monetary Models

The Evidence

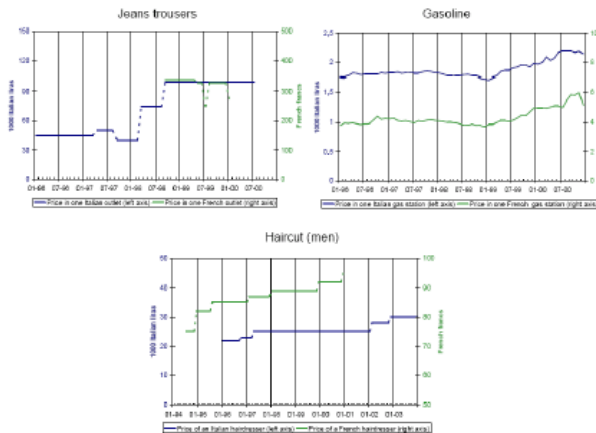
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

The Evidence

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry et al. (2004) and Yeronese et al. (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhvne et al. WP 05

Why New Keynesian?

- ▶ Real Rigidities - Monopolistic Competition
- ▶ Nominal Rigidities - Calvo Pricing
- ▶ But with Microfoundations
 - ▶ Method RBC literature (Dynamic Stochastic General Equilibrium)
 - ▶ Explain Persistence and Demand Shocks

The Basic Macro Structure

The Basic Structure of DSGE Models

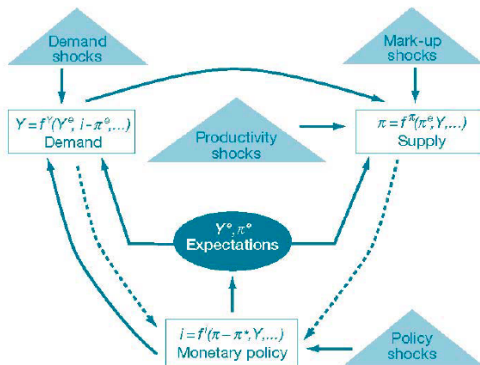


Figure 1. The basic structure of DSGE models. Source: Sbordone *et al.* (2010).

The Simple Macro Structure

Supply (NK Phillips Curve)

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \quad (1)$$

Demand (NK IS)

$$y_t = E_t y_{t+1} - \gamma (i_t - E_t \pi_{t+1}) + \eta_t \quad (2)$$

Monetary Policy Rule (Taylor Rule)

$$i_t = \bar{r}_t + \phi_\pi (\pi_t - \pi^*) + \phi_y (y_t) \quad (3)$$

Where Do They Come From?

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \quad (4)$$

Derived from optimal pricing behaviour of the firm with market power and nominal stickiness (Calvo Pricing)

$$y_t = E_t y_{t+1} - \gamma (i_t - E_t \pi_{t+1}) + \eta_t \quad (5)$$

Directly from the first order condition of the consumer and equilibrium conditions

$$i_t = \bar{r}_t + \phi_\pi (\pi_t - \pi^*) + \phi_y (y_t) \quad (6)$$

Monetary policy reaction function

Note:

- ▶ There is no need of having an LM (no explicit money)
- ▶ Both π_t and y_t are jumping variables (function of expectations of future state variables)

Consumer Problem

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right] \quad (7)$$

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (8)$$

$$P_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \quad (9)$$

$$P_t C_t + B_t = W_t N_t + (1 + r_t) B_{t-1} + \Pi_t \quad (10)$$

Consumer Problem

Optimal Allocation of Consumption Expenditure

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj \quad (11)$$

subject to

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (12)$$

Optimal Allocation

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t \quad (13)$$

Consumer Problem

Optimal Dynamic Consumption/Leisure Decision

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right] \quad (14)$$

$$C_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + (1 + r_t) \frac{B_{t-1}}{P_t} + \frac{\Pi_t}{P_t} \quad (15)$$

Lagrangian

$$L = E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right] + \quad (16)$$

$$+ E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \left[\frac{W_{t+i}}{P_{t+i}} N_{t+i} + (1 + r_{t+i}) \frac{B_{t+i-1}}{P_{t+i}} + \frac{\Pi_{t+i}}{P_{t+i}} - C_{t+i} - \frac{B_{t+i}}{P_{t+i}} \right] \quad (17)$$

Consumer Problem

Optimal Dynamic Consumption/Leisure Decision

FOC

$$\frac{\partial L}{\partial C_t} = C_t^{-\sigma} + \lambda_{t+i} = 0 \quad (18)$$

$$\frac{\partial L}{\partial N_t} = -\chi N_t^\eta + \lambda_t \frac{W_t}{P_t} = 0 \quad (19)$$

$$\frac{\partial L}{\partial B_t} = -\lambda_t \frac{1}{P_t} + E_t \left[\beta \lambda_{t+1} (1 + r_{t+1}) \frac{1}{P_{t+i}} \right] = 0 \quad (20)$$

Consumer Problem

Optimal Dynamic Consumption/Leisure Decision

Rearranging and using condition (18) to eliminate the Lagrange multiplier, we get:

$$\chi N_t^\eta = C_t^{-\sigma} \frac{W_t}{P_t} \quad (21)$$

$$C_t^{-\sigma} = E_t \left[\beta (1 + r_{t+1}) \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma} \right] \quad (22)$$

As shown before, this implies the following log linear relationships

$$w_t - p_t = \sigma c_t + \eta n_t \quad (23)$$

$$c_t = E_t \{ c_{t+1} \} - \frac{1}{\sigma} \{ i_t - E_t \pi_{t+1} - \ln \beta \} \quad (24)$$

Firm Problem

Calvo Pricing

- ▶ A firm may change price of its product only when Calvo Fairy visits.
- ▶ The probability of a visit is $(1 - \theta)$
- ▶ It is independent of the length of the time and the time elapsed since the last adjustment. Hence, in each period the $(1 - \theta)$ share of firms
- ▶ may change their price and rest, θ , keep their price unchanged.
- ▶ Mathematically, Calvo Fairy's visits follows Bernoulli process (discrete version of Poisson process).
- ▶ The probability distribution of the number of periods between the visits of Calvo Fairy is geometric distribution.
- ▶ The expected value of geometric distribution and, hence, the average number of periods between the price changes (of a firm) is

$$\frac{1}{1 - \theta}$$

Firm Problem

Calvo Pricing

Price Stickiness: Firms can change prices with a probability $(1 - \theta)$

Price Decision - Intertemporal

Problem of the Firm (when they can change the prices)

$$\min_{z_t} L(z_t) = \sum_{k=0}^{\infty} (\theta\beta)^k E_t (z_t - p_{t+k}^*)^2 \quad (25)$$

$$L'(z_t) = 2 \sum_{k=0}^{\infty} (\theta\beta)^k E_t(z_t - p_{t+k}^*) = 0 \quad (26)$$

$$\left[\sum_{k=0}^{\infty} (\theta\beta)^k \right] z_t = \sum_{k=0}^{\infty} (\theta\beta)^k E_t(p_{t+k}^*) \quad (27)$$

$$\frac{z_t}{1 - \theta\beta} = \sum_{k=0}^{\infty} (\theta\beta)^k E_t(p_{t+k}^*) \quad (28)$$

$$z_t = (1 - \theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t(p_{t+k}^*) \quad (29)$$

Reset Price

$$z_t = (1 - \theta\beta) p_t^* + \theta\beta E_t z_{t+1} \quad (30)$$

where

$$E_t z_{t+1} = \sum_{k=0}^{\infty} (\theta\beta)^k E_t (p_{t+1+k}^*)$$

"frictionless optimal" price p^* .

$$p_t^* = \mu + mc_t \quad (31)$$

Thus the reset price can be written as:

$$z_t = (1 - \theta\beta) (\mu + mc_{t+k}) + (1 - \theta\beta) E_t z_{t+1} \quad (32)$$

Aggregate Pricing

$$p_t = \theta p_{t-1} + (1 - \theta) z_t \quad (33)$$

This can be rearranged as

$$z_t = \frac{1}{1 - \theta} (p_t - \theta p_{t-1}) \quad (34)$$

$$\frac{1}{1 - \theta} (p_t - \theta p_{t-1}) = \frac{\theta \beta}{1 - \theta} (p_{t+1} - \theta p_t) + (1 - \theta \beta) (\mu + mc_t)$$

this equation implies:

$$\pi_t = \beta \pi_{t+1} + \frac{1 - \theta}{\theta} (1 - \theta \beta) (\mu + mc_t - p_t) \quad (35)$$

New Keynesian Phillips Curve

Denoting $\widehat{mc}_t = \mu + mc_t - p_t$

$$\pi_t = \beta\pi_{t+1} + \frac{1-\theta}{\theta} (1-\theta\beta) \widehat{mc}_t \quad (36)$$

$$\widehat{mc}_t = \lambda y_t$$

$$\pi_t = \beta\pi_{t+1} + \gamma y_t \quad (37)$$

where

$$\gamma = \frac{\lambda (1-\theta) (1-\theta\beta)}{\theta}$$

Inflation as an Asset Price

Solving Forward the New Keynesian Phillips curve (as for the Cagan Model)

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t y_{t+k}$$

Problems

- ▶ Not enough dynamic in the model
- ▶ Any change will be reflected instantaneously on the variables
- ▶ Many attempts to produce more sluggish response (will see later)

System Stability

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \quad (38)$$

$$y_t = E_t y_{t+1} - \gamma (i_t - E_t \pi_{t+1}) + \eta_t \quad (39)$$

:

$$i_t = \bar{i}_t + \phi_\pi (\pi_t - \pi^*) + \phi_y (y_t) \quad (40)$$

Where $\phi_\pi, \phi_y \geq 0$.

System Stability

State Space Representation

Substituting the general (40) now in the above system can be written in state-space form:

$$E_t \mathbf{z}_{t+1} = \tilde{\mathbf{A}} * \mathbf{z}_t + \tilde{\mathbf{B}} * \pi^* + \mathbf{K} \mathbf{u}_{t+1}$$

where, $\mathbf{z}'_t = \begin{bmatrix} \pi_t & y_t \end{bmatrix}$, $\mathbf{u}'_t = \begin{bmatrix} \varepsilon_t & \eta_t & \bar{i}_t \end{bmatrix}$ and

$$\tilde{\mathbf{A}} = \begin{pmatrix} \frac{1}{\beta} & -\frac{\alpha}{\beta} \\ -\frac{\gamma}{\beta} + \gamma\phi_\pi & \frac{\alpha\gamma}{\beta} + 1 + \gamma\phi_y \end{pmatrix}$$

Stability of the system eigenvalues of matrix $\mathbf{A}$ outside the unit circle (Blanchard and Kahn, 1980).

System Stability

Stability Condition

This implies the following conditions

$$\left| \frac{tr(\tilde{\mathbf{A}}) \pm \sqrt{tr(\tilde{\mathbf{A}})^2 - 4 \det(\tilde{\mathbf{A}})}}{2} \right| > 1$$

which, given our parameter restrictions and the nature of the problem, reduces to the following three conditions (See Woodford 2003 for detailed derivation)

$$\det(\tilde{\mathbf{A}}) > 0, \quad \det(\tilde{\mathbf{A}}) - tr(\tilde{\mathbf{A}}) > -1, \quad \det(\tilde{\mathbf{A}}) + tr(\tilde{\mathbf{A}}) > -1$$

System Stability

Stability Condition

Necessary Condition For Stability

$$\left(\frac{1-\beta}{\alpha}\right)\phi_y + \phi_\pi > 1 \quad (41)$$

Sufficient Condition

$$\phi_\pi > 1 \quad (42)$$

Taylor Rule: for the system to be stable the CB should react with an elasticity of interest rate relative to inflation greater than one (Taylor proposed $\phi_\pi = 1.5$ and $\phi_y = 0.5$)