The Emergence of Weak, Despotic and Inclusive States*

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Abstract

Societies under similar geographic and economic conditions and subject to similar external influences nonetheless develop very different types of states. At one extreme are weak states with little capacity and ability to regulate economic or social relations. At the other are despotic states which dominate civil society. Yet there are others which are locked into an ongoing competition with civil society and it is these, not the despotic ones, that develop the greatest capacity. We develop a model of political competition between state (controlled by a ruler or a group of elites) and civil society (representing non-elite citizens), where both players can invest to increase their power. The model leads to different types of steady states depending on initial conditions. One type of steady state, corresponding to a weak state, emerges when civil society is strong relative to the state (e.g., having developed social norms limiting political hierarchy). Another type of steady state, corresponding to a despotic state, originates from initial conditions where the state is powerful and civil society is weak. A third type of steady state, which we refer to as an inclusive state, emerges when state and civil society are more evenly matched. In this last case, each party has greater incentives to invest to keep up with the other, which undergirds the rise of high-capacity states and societies. Our framework highlights that comparative statics with respect to structural factors such as geography, economic conditions or external threats, are *conditional* — in the sense that depending on initial conditions they can shift a society into or out of the basin of attraction of the inclusive state.

Keywords: civil society, contest, political divergence, state capacity, weak states.

JEL classification: H4, H7, P16.

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1 Introduction

The capacity of the state to enforce laws, provide public services, and regulate and tax economic activity varies enormously across countries. The dominant paradigm in social science to explain the development of state capacity links this diversity to the ability of a powerful group, elite or charismatic leader to dominate other powerful actors in society and build institutions such as a fiscal system or bureaucracy (e.g., Huntington, 1968). This paradigm also relates this ability to certain structural factors such as geography, ecology, natural resources and population density (Mahdavy, 1970, Diamond, 1997, Herbst, 2000, Fukuyama, 2011, 2014), the threat of war (Hintze, 1975, Brewer, 1989, Tilly, 1975, 1990, Besley and Persson, 2009, 2011, O'Brien, 2011, Gennaioli and Voth, 2015), or the nature of economic activity (Mann, 1986, 1993, Acemoglu, 2005, Spruyt, 2009, Besley and Persson, 2011, Mayshar, Moav and Neeman, 2011). However, historically, societies with similar ecologies, geographies, initial economic structures and external threats have diverged sharply in terms of the development of their states. Also strikingly, in many instances in which a state builds considerable capacity, it does not do so by dominating a meek society; on the contrary, state capacity often appears to be called into existence by the demands of society and regular citizens, and we see it develop most consistently when states and elites controlling state institutions have to contend and struggle with a strong, assertive (civil) society.[1]

The role of society, not just of the state and elites, in the emergence of state capacity is illustrated by the historical evolution of the power of the state and state-society relations in Europe. This is best documented in the English case where local communities led the way in organizing law enforcement, public good provision and conflict resolution in the early modern and modern periods (Braddick, 2000, and Hindle, 2002, for overviews).[2] Policy-setting in this period thus "appears more like a dynamic process of communication between center and localities rather than a one-sided drive towards ever greater penetration or acculturation" (Kümin and Wurgler, 1997, p. 40). Harris (1993, p. 33) goes even further and argues that government "was moulded more by pressures from within political society than by the efforts of kings or officials to direct it from above". Though initiatives did emanate from the state as well (e.g., Elton, 1952), Hindle (2002, p. 16) shows that "The early modern state did not become more active at the expense of society; rather it did so as a consequence of social need".[3] Indeed, many significant policy initiatives, such as the Elizabethan Poor Laws, came not from the central state but from local society, in this case Norwich (see Braddick, 2000, Chapter 3). Notably, there was a two-way feedback from society to state and back to society. The growth of the state "drew together provincial communities into a more closely integrated national society", and "introduced a new depth and complexity to their local patterns of social stratification" (Wrightson 1982, pp. 222-3). State expansion changed society which led to further demands (Tilly, 1995).[4]

[1]Throughout, we use "civil society" and "society" interchangeably to represent the "non-elites", meaning regular citizens and non-governmental organizations.

[2]The remarkable development of English state capacity during this period is illustrated by Goldie's (2005) calculation that there were around 50,000 parish officers in 1700, or around 5% of adult male population. In 1800 this number was 100,000. Since there was constant rotation of offices the number of people who had held office in England was much larger.

[3]This is particularly the case of the law where individual demands for legal services and popular participation are well documented see Herrup (1989) and Brooks (2009).

[4]The English pattern is representative of the broader European experience, see the essays in Blickle ed. (1997) and

European history also illustrates the divergent paths of state-society relations starting from a great deal of shared history, culture and economic fundamentals. At one end of the spectrum, Prussia from the 17th century constructed an autocratic, militarized state under an absolutist monarchy, backed by a traditional landowning Junker class, which continued to exercise enough authority to help derail the Weimar democracy in the 1920s (Evans, 2005). Meanwhile, just to its south, the Swiss state attained its final institutionalization in 1848, not as a consequence of an absolutist monarchy, but from the bottom-up construction of independent republican cantons based on rural-urban communes. A little further south, places in the Balkans, such as Montenegro and Albania, never developed any centralized state authority. Prior to 1852 Montenegro was in effect a theocracy, but its ruling Bishop, the Vladika, could exercise no coercive authority over the clans which dominated the society partly via a complex web of traditions and social norms. As a consequence of this lack of state authority, blood feuds and other inter-clan conflicts were extremely common.

This diversity is not associated with large structural differences. Switzerland and Montenegro are both mountainous (which Braudel, 1966, emphasized as crucial), were both part of the Roman Empire, have been Christian for centuries, were specialized in similar economic activities such as herding, and have been involved in continuous wars against external foes. Before the founding of the Swiss Confederation in 1291, feuding was also common in that area. Scott, for example, notes: "There is general agreement amongst recent historians that the origins of the Swiss Confederation lay in the search for public order. The provisions of the *Bundesbrief* of 1291 were clearly directed against feuding in the inner cantons" (1995, p. 98; see also Blickle, 1992). The parallels between Switzerland and Prussia are even stronger. Both countries have very close cultural and ethnic roots (and historically Switzerland had been settled by Germanic tribes, particularly the Alemanni), and have shared similar religious identities before and after the Reformation. The core of Prussia, Brandenburg, like Switzerland, was part of the Holy Roman Empire, and had feudal roots similar to those of Switzerland.

In this paper, we develop a simple theory of state-society relations where the competition and conflict between state and (civil) society is the main driver of the institutional change and the emergence of state capacity, and crucially, state capacity develops most powerfully when the state (and elites) are evenly matched with society (non-elites). Though the elites that control the state wish to establish dominance over society, the ability of society to develop its own strengths (in the form of coordination, social norms and local organization) is central, because it induces the state to become even stronger in order to compete with society. When this balance between state and society is not achieved, either the state fully dominates society or society is powerful and the state remains weak. Crucially, however, when society is weak, state capacity is relatively limited as well, because the state can control society easily and does not need to invest much in its own capacity.[5] In addition to highlighting the vital role of the race between state and society in the development of state capacity,

Blockmans, Holenstein, Mathieu and Schläppi eds. (2009) for an introduction. Habermas (1989) in his famous work on the origins of the 'public sphere' viewed this as the outcome of state formation, arguing that "Civil society came into existence as the corollary of a depersonalized state authority" (1989, p. 19)

[5]Throughout, we use power, strength and capacity interchangeably. In practice, one might wish to distinguish between the underlying, infrastructural power of the state, which then creates capacity to achieve certain goals or implement certain objectives, but in our abstract model, this distinction does not arise.

our theory shows that, just as in the examples discussed above, polities with similar initial conditions and subject to similar structural influences can nonetheless experience divergent state-society relations and evolution of state capacity, because they may fall into the basins of attraction of different dynamic equilibria.

This history-dependent development of state capacity and our main theoretical results are summarized in Figure 1. This figure plots the global dynamics of state-society relations. Region I illustrates the Huntingtonian path, approximating the political dynamics of Prussia. Here, the state is stronger than civil society to start with and fully dominates it; for this reason, we call this type of state *despotic*. Region III is the case in which the society's norms, especially in how non-elites are able to act collectively and control political power and hierarchy, are strong and this prevents the emergence of a powerful state, paving the way to *weak states* as in Montenegro. Region II illustrates the happy middle ground where state and society are initially in balance, and this triggers an ongoing competition between the two, whereby they both become stronger over time. We refer to this type of state, which best resembles the English or Swiss path, as the *inclusive state*.[6] As the figure shows, it is in this inclusive case that the state achieves the greatest capacity. The fact that the capacity of the state is greater in this case than in Region I highlights that it is the competition between state and society that triggers greater investments by the state (or the ruler and elites controlling it).

One attractive feature of our conceptual framework is already visible in Figure 1. It suggests that the effects of a rich array of structural factors are *conditional*. For example, factors that (exogenously) increase the power of the state could move a polity from Region III to Region II, initiating a powerful process of state capacity building. But the same factors may also push a society previously in Region II into Region I, reducing its long-run potential to achieve high state capacity. Similarly, structural factors also shift the boundaries of the basins of attraction of the three different types of states, but the effects of such changes are also conditional on the prevailing balance between state and society. These conditional comparative statics thus provide one explanation for why for each structural factor argued to underpin the development of state capacity, there are always several counterexamples going in the opposite direction (see, e.g., Hoffman, 2015, on this).

Our theory and Figure 1 also provide a reinterpretation of the divergent paths of Prussia, Switzerland and Montenegro, even though there appear to be no large structural differences affecting these nations. Rather, there were small differences favoring the development of a powerful state in Prussia, strengthening society at the expense of the state in Montenegro, and pitting the two more equally against each other in Switzerland. The Prussian state fused Brandenburg with the legacy of the militarized state of the Teutonic Knights to the east of the River Elbe, where feudalism was possibly the most intense in Europe (Gerschenkron, 1943, Moore, 1966, Clark, 2009). In contrast, conditions likely favored the weak state path in Montenegro, where the 'herdsman' culture was very strong and a more centralized political order like the Holy Roman Empire was absent in the Middle Ages. In contradistinction to both of these cases, the powers of state and society were more evenly balanced in Switzerland. Differently from Prussia, Swiss peasants were more 'free' (Steinberg, 2015), independent

[6] This terminology is motivated by the fact that, in this case, the state is not just strong (capable) but also evenly matched with civil society, which is then able to actively participate in political conflict and partially check the domination of the state and the elites that control it.

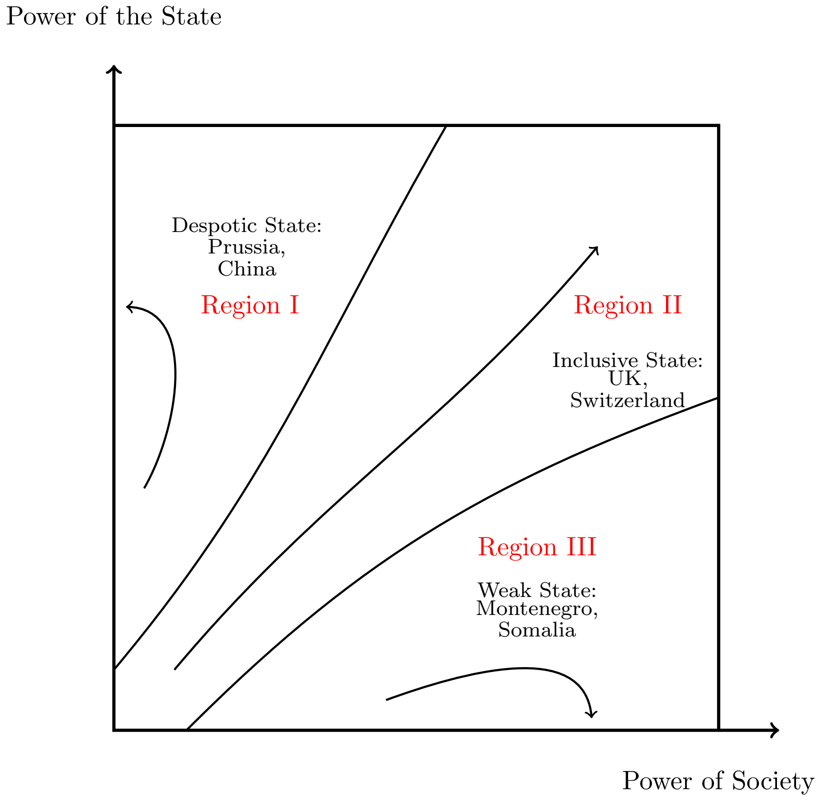

Figure 1: The emergence and dynamics of weak, despotic and inclusive states.

cities such as Basel, Bern and Zürich played a more important economic and political role, and the major demographic changes of the 14th century, in particular the Black Death of the 1340s, appear to have weakened the elites even further (e.g., Morerod and Favrod, 2014). Compared to Montenegro, Switzerland's history of established political order under the Holy Roman Empire and of corporations such as monasteries and cathedral chapters (Church and Head, 2013, Morerod and Favrod, 2014) may have created the small differences facilitating the emergence of a state capable of competing against civil society.

Theoretically, our setup is one of a dynamic contest between two players, the elite controlling the state and the (civil) society representing non-elite citizens. At each date, the state and society both choose investments in their strength, and these strengths determine both the overall output in the economy which is distributed between the elite controlling the state and the rest of the citizens, and how this distribution takes place. We introduce some degree of economies of scale in the contest technology so that the cost of investment for either state or society becomes higher if their strength falls below a certain level.[7] The interplay of contest incentives and the presence of economies of scale underpins the emergence of three stable steady states as shown in Figure 1: when one party is significantly stronger than the other, the weaker player is discouraged from investing. But since as

[7] We view this as a reasonable assumption both for society and the state. Many scholars have argued that there are increasing returns to collective action (e.g., Marwell and Oliver, 1993, Pearson, 2000) or the acquisition of social capital (Francois, 2002). It also appears reasonable that there are fixed costs involved in the creation of fiscal systems or bureaucracies (e.g. Dharmapalaa, Slemrod and Wilson, 2011, Gauthier, 2013).

in contest models, part of the reason why each player invests is to be stronger than the other, the discouragement of the weaker party also reduces the investment incentives of the dominant player. In contrast, when the two players are evenly matched, they are both induced to invest more. These results are an application of Harris and Vickers' (1984, 1987) *discouragement effect*.[8] After illustrating the workings of our model in the simplest possible case where the players act myopically, we show that similar results obtain when the players are forward-looking but sufficiently impatient.

Our model is purposefully reduced-form and simple, which helps to highlight the potential generality of the forces we are emphasizing. One drawback of this strategy is that the mapping from the investments by the state and society in our model to the data is not as clear as it would have been under a setup with explicit microstructure. We attempt to partially rectify this in Section 7 by explaining how state-society relations and the divergent paths of political development in Switzerland, Prussia and Montenegro can be understood through the lenses of our model.

Our paper is related to a number of literatures. As already discussed above, prominent in the social science literature on state building are approaches that situate the roots of state capacity in the ability of the state and groups controlling it to dominate society.[9] In addition, these approaches also emphasize the role of structural factors in triggering or preventing state building. Our theory thus sharply differs from these approaches, and has much more in common with a few works in sociology and political science emphasizing the interaction of state and society. Most importantly, Migdal (1988, 2001) argues that weak states are a consequence of a strong society (as in our Region III). Scott (2010) has similarly stressed the ability of people to resist the state and its interference. Putnam (1993) argued that a strong society leads to better governance and bureaucratic effectiveness. None of these scholars note our key distinction from the previous literature — the idea that state capacity develops most strongly when state and civil society are matched in terms of their strengths and compete dynamically. Acemoglu (2005) argues that the capacity of the state is highest when it is "consensually strong". In his model this emerges not because of competition between state and society, but as a result of a repeated game equilibrium in which citizens are expected to replace rulers who do not provide sufficient public goods or otherwise misbehave.

The distinction between our approach and the literature is well illustrated by the difference between the three-way taxonomy we propose and that of Besley and Persson (2014). In their theory elites decide on the strength of the state, and depending on structural differences, how elites may end up choosing different sorts of states (which they call "common interest", "special interest" and "weak"). In contrast to this approach, in our framework societies develop different types of states

[8]See Dechenaux, Kovenock and Sheremeta (2015) for a survey of experimental evidence on discouragement effect in contests. See also Aghion, Bloom, Blundell, Griffith and Howitt (2005) and Aghion and Griffith (2008) for evidence on the discouragement effect in the context of innovation investments.

[9]In addition to the works such as Huntington (1968), Tilly (1990) and Fukuyama (2011) mentioned above, this includes authors emphasizing the role of state capacity in enabling elites controlling the state to dominate society via various means, including repression (e.g., Anderson, 1974, Hechter and Brustein, 1980, Slater, 2010, Saylor, 2014).

The recent economics literature, mentioned above, mirrors these approaches. For example, Besley and Persson (2009, 2011) focus on the incentives of the elites controlling the state to undertake investments to build state capacity and link this to the probability that they will lose power domestically and to external threats. Gennaioli and Voth (2015) develop a model of the interaction between warfare and state capacity, while Mayshar, Moav and Neeman (2011) emphasize the effect of the type of crop on state building. Other work by Acemoglu, Robinson and Santos-Villagran (2013) and Acemoglu, Ticchi and Vindigni (2011) again emphasize elite incentives. Acemoglu, Robinson and Torvik (2016) study more systematically the interaction between elite incentives and social mobilization.

not primarily because of the choices of elites, but as a consequence of the struggle between elites and society, and small differences in the balance of power in this struggle can cause nations to be in the basins of attraction of very different long-run patterns of political development.

Our work is also related to a large literature in archaeology focusing on how societies start the process of state formation (so-called "pristine state formation"). Most of these, for example Flannery (1999) or Flannery and Marcus (2013), emphasize a 'top-down' elite centric approach, but other work, particularly by Blanton and Fargher (2008), has placed equal weight on the role of society.

Finally, our model is an example of a dynamic contest, though most of our analysis involves myopic players. Static models of contests in economics go back at least to Tullock (1980), and have been more systematically studied by Dixit (1987), Skaperdas (1992, 1996), Cornes and Hartley (2005), and Corchon (2007). They are similar to models of (patent) races as in Loury (1979), and to all-pay auctions as studied, among others, by Baye et al. (1996), Krishna and Morgan (1997) and Siegel (2009). Our formulation uses a contest function in differences, introduced by Hirshleifer (1989), and is mathematically closer to all-pay auctions (e.g., Che and Gale, 2000). Dynamic contests and related racing models are more challenging and various special cases have been discussed in Fudenberg et al. (1983), Harris and Vickers (1985, 1987), Grossman and Shapiro (1987), Grossman and Kim (1996), Konrad (2009, 2012), and Cao (2014), while the literature on dynamic public good games deals with related problems, but crucially without the contest element (e.g., Levhari and Mirman, 1980, Lockwood and Thomas, 2002, and Battaglini, Nunnari and Palfrey, 2014). None of these papers, and to the best of our knowledge no others, derive the possibility of three locally stable steady states with different configurations of power.[10]

The rest of the paper is organized as follows. In the next section, we introduce our main model. In Section 3, we characterize the dynamic equilibrium and steady states of this model when players are short-lived or myopic. To maximize transparency, this section uses a number of simplifying assumptions, many of which are relaxed later. Section 4 analyzes the same model with forward-looking players, and establishes that the same results when these players are sufficiently impatient. Section 5 relaxes one of the most important simplifying assumptions, allowing the investments of the state and civil society to also affect the size of the pie to be divided. In this setup, it also provides additional comparative static results on how different steady states and their basins of attraction are affected by changes in parameters. Section 7 discusses the historical evidence relevant to the interpretation of our model. Finally, Section 8 concludes, while the Appendix provides some generalizations and microfoundations for the setup studied in the main text.

2 Basic Model

In this section, we introduce our basic model. We start with an overview of the main elements of the model and also provide a justification for our overall approach and modeling assumptions. The model is then analyzed in the next several sections.

[10] In a very different context, Benabou, Ticchi and Vindigni (2015) emphasize how the initial balance of power between religion and science can influence the long-run evolution of a society.

2.1 Overview, Motivation and Interpretation

As already highlighted in the Introduction, our approach is motivated by four sets of empirical regularities and our formal model is designed to focus on these regularities in the most parsimonious manner. These are:

1. *Three different types of states:* Weak, despotic and inclusive states — with very different state-society relationships — can emerge starting from similar initial conditions. Examples of all three types of states originating from relatively similar initial conditions are pervasive in history, and we illustrate this in the context of a specific setting in Section 7. To shed light on the mechanisms that can lead to three different types of steady states with different implications for state capacity is one of the main objectives of our formal model.

2. *Sources of state capacity:* The more conventional distinction in the literature is between weak and strong states, and the most common approach tends to emphasize the emergence of state capacity from the ability of a leader or a group to dominate the rest and establish a type of monopoly of power (e.g., Huntington, 1968). As already discussed in the Introduction, the historical record points to a very different process. This evidence suggests that, in sharp contrast to the implications of the approach by Huntington and his followers, state capacity expands most powerfully when non-elites ("society") can contest the power of the state and elites. Formalizing this notion and explicating why it makes sense is another one of our objectives.

3. *Power of society:* The source of the power of non-elites against the better organized and richer elites and state institutions typically rests on coordination or "collective action". Such coordination can be achieved by norms or can take a more institutionalized form when it is orchestrated by organizations (such as non-government organizations and political parties) and is mediated by political institutions (such as voting or societal oversight). The purpose of such coordinated action is often to limit or contain the power of the elite. This is most apparent in small-scale societies where many norms are targeted at limiting political hierarchy and inequality (e.g., Boehm, 2001, Flannery and Marcus, 2013). Similarly, a prime objective of institutions and organizations coordinating the participation of non-elites in politics is to limit the power of and potential abuses by political elites and state institutions.

4. *The role of structural factors:* While much of the social science literature on state-building and the origins of state capacity emphasizes structural factors (e.g., population, geography, culture, threat to war, the types of crops, the technology of war-making, etc.), our discussion in Section 7 reveals that the influence of these factors, if any, is much more complex than what the previous literature maintains. The same factors appear to sometimes facilitate the process of state-building and sometime hinder it (as pointed out also in Hoffman, 2015, for the effects of warfare). Shedding light on these possibilities — and developing a unified framework in which such structural factors can have an impact but this impact is "conditional" on prevailing conditions — is another one of our major objectives.

We design our model to capture these issues in the simplest possible fashion. Though reality is undoubtedly much more complex, especially in terms of multi-faceted heterogeneities, the most parsimonious approach for our purpose is to consider the competition between state and society, that is, between elites and state institutions (typically but not always controlled by elites) on the one hand and non-elites on the other. This approach, of course, greatly simplifies by eschewing the modeling of interesting and consequential heterogeneities between elites (for example, between monarchs and lords as in the context of medieval European history) and within society itself (for example, differences by income and other identities). A partial justification for our approach is that, despite its simplicity, it delivers new insights on the main objectives outlined above, and opens the way for the study of richer models in the future.

Our model is also simplified by assuming that both the power of the state and the power of society are one-dimensional. This is again motivated by tractability. The power of the state, denoted by s_t below, best approximates what Mann (1986) terms "infrastructural power" — capturing the presence of the state and its capacity to achieve a range of objectives. As Mann himself emphasizes, this differs from other dimensions of the state, including its military, judicial or bureaucratic powers. Imposing that the power of society, denoted by x_t in the model, is one-dimensional further means that we are not modeling how different types of societal coordination interact with political objectives. Moreover, throughout, we assume that the power of society accumulates as a result of investments just like the power of the state. This modeling choice is motivated by our desire to keep the model parsimonious while also highlighting that the specific microfoundations of the power of society are not crucial for the dynamics we want to highlight here. For example, while political organizations evolve as a result of investments by individuals or leaders, social norms that play the most major role of political coordination in small-scale societies tend to evolve slowly over time, but most likely still respond to some of the political exigencies, such as the need to control chiefs or "big men" (e.g., Boyd and Richerson, 1988). Our reduced-form modeling ignores the distinctions, but we believe it captures the important point that, in order to contain political inequality, the coordination of society will need to increase when the power of the elite or state institutions increase.

Finally, though both the power of the state and that of society create a range of benefits, we also adopt a reduced-form approach in modeling these and assume that aggregate surplus at a point in time is a function of these powers. In reality, the ability of society to coordinate and the infrastructural power of the state contribute to the productivity of producers, which then affects the surplus to be divided between different parties. We show at the end of the section that when these aspects are modeled in greater detail, the reduced-form representation we are introducing next can be obtained as part of the equilibrium.

2.2 Preferences and Conflict

We start with a discrete time setup, where period length is $\Delta > 0$ and will later be taken to be small, so that we work with differential rather than difference equations in characterizing the dynamics. At time t, the state variables inherited from the previous period are $(x_{t-\Delta}, s_{t-\Delta}) \in [0, 1]^2$, where the first element corresponds to the strength of civil society and the second to the strength of the state controlled by the elite.

At each point in time, the elite or the state is represented by a single player, and civil society is also represented by a single player. In the next two sections, we study both the case in which these players are short-lived and are immediately replaced by another player (so that we have a non-overlapping generations model with "myopic" players), and the case in which players are long-lived and maximize their discounted sum of utilities.

At time t, players simultaneously choose their investments, $i_t^x \geq 0$ and $i_t^s \geq 0$, which determine their current strengths according to the equations:

$$x_t = x_{t-\Delta} + i_t^x \Delta - \delta \Delta, \tag{1}$$

and

$$s_t = s_{t-\Delta} + i_t^s \Delta - \delta \Delta, \tag{2}$$

where $\delta > 0$ is the depreciation of the strength of both parties between periods. Both investment and depreciation are multiplied by the period length, Δ, since they represent "flow" variables, and when period length is taken to be small, they will be suitably downscaled.[11]

The cost of investment for civil society during a period of length Δ is given as $\Delta \cdot \tilde{C}_x(i_t^x, x_{t-\Delta})$ where

$$\tilde{C}_x(i_t^x, x_{t-\Delta}) = \begin{cases} c_x(i_t^x) & \text{if } x_{t-\Delta} > \gamma_x, \\ c_x(i_t^x) + (\gamma_x - x_{t-\Delta})\, i_t^x & \text{if } x_{t-\Delta} \leq \gamma_x. \end{cases}$$

This cost function is multiplied by Δ, since it is the cost of investing an amount i_t^x during the period of length Δ (as captured by equation (1)). The presence of the term $\gamma_x > 0$, on the other hand, captures the "increasing returns" nature of conflict mentioned in the Introduction: starting from a low level of conflict capacity, it is more costly to build up this capacity. We specify this in a very simple form here, with the cost of investments increasing linearly as last period's conflict capacity falls below the threshold γ_x. This increasing returns aspect plays an important role in our analysis as we emphasize below.

The cost of investment for the state during a period of length Δ is similarly given as $\Delta \cdot \tilde{C}_s(i_t^s, s_{t-\Delta})$ where

$$\tilde{C}_s(i_t^s, s_{t-\Delta}) = \begin{cases} c_s(i_t^s) & \text{if } s_{t-\Delta} > \gamma_s, \\ c_s(i_t^s) + (\gamma_s - s_{t-\Delta}) i_t^s & \text{if } s_{t-\Delta} \leq \gamma_s. \end{cases}$$

In these expressions, it will often be more convenient to eliminate investment levels and directly work with the two state variables, x_t and s_t, especially when we take Δ to be small and transition to continuous time. In preparation for this transition, let us substitute out the investment levels and observe that the cost function for civil society and state can be written as:

$$C_x(x_t, x_{t-\Delta}) = c_x\left(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta\right) + \max\{\gamma_x - x_{t-\Delta}, 0\}\left(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta\right),$$

and

$$C_s(s_t, s_{t-\Delta}) = c_s\left(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta\right) + \max\{\gamma_s - s_{t-\Delta}, 0\}\left(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta\right),$$

where the increasing returns to scale nature of the cost function is now captured by the max term.[12]

[11] Assuming that depreciation is independent of the current level of the strength of the state or civil society is for convenience only. In addition, we can easily allow the two state variables to have different depreciation rates, but do not do so in order to prevent the notation from becoming more cumbersome.

[12] Note that when we consider the limit $\Delta \to 0$, we obtain

$$\begin{aligned} C_x(\dot{x}_t) &= c_x(\dot{x}_t + \delta) + \max\{\gamma_x - x_t, 0\}(\dot{x}_t + \delta), \\ C_s(\dot{s}_t) &= c_s(\dot{s}_t + \delta) + \max\{\gamma_s - s_t, 0\}(\dot{s}_t + \delta). \end{aligned}$$

During the lifetime of each generation, a polity with state strength s_t and civil society strength x_t produces output/surplus given by

$$f(x_t, s_t), \tag{3}$$

where f is assumed to be nondecreasing and differentiable.[13] The dependence of the total output of the economy on the strength of the state captures the various efficiency-enhancing roles of state capacity. In addition, we allow for output to depend on the strength of civil society as well, which might be because a strong civil society prevents extractive uses of the capacity of the state that tend to reduce the total output or surplus in the economy, or because its greater cooperation and coordination improves economic efficiency (see Section 2.4).

We next discuss how the output of society is distributed between the elite (controlling the state) and citizens. At date t, if the elite and civil society (citizens) decide to fight, then one side will win and capture all of the output of the economy, and the other side receives zero. Winning probabilities are functions of relative strengths. In particular, the elite will win if

$$s_t \geq x_t + \sigma_t, \tag{4}$$

where σ_t is drawn from the distribution H independently of all past events. We denote the density of the distribution function H by h. The existence of the random term σ_t captures the fact that various stochastic factors impact the outcome of any conflict. Throughout, since both sides have the same assessment of the outcome of conflict, we will presume that they divide total output according to their expected shares, but whether they do so or actually engage in conflict is immaterial for our results.

This specification of the stochastic contest function, and a symmetry assumption which we will shortly impose, implies that when the strengths of civil society and state are given, respectively, by x and s, the probability that the state will win the conflict is $H(s - x)$, and the probability that the civil society will do so is $1 - H(s - x) = H(x - s)$, a property we will use frequently below.

In the Appendix, we also show that the most important qualitative features implied by this formulations of conflict between the elite (state) and society are shared by other formulation of the contest between these parties.

2.3 Assumptions

We next introduce three assumptions. The first one is a simplifying assumption, which we impose initially and then relax subsequently:

Assumption 1 $f(x, s) = 1$ for all $x \in [0, 1]$ and $s \in [0, 1]$.

This assumption makes it transparent that the multiple steady-state equilibria and their dynamics — our main focus — are driven by the dynamic contest between the state and civil society, not because of changes in the value of the prize in this contest. It will be relaxed in Section 5.

[13] The fact that (3) refers to output during the lifetime of each generation means that each generation will produce this quantity regardless of $\Delta > 0$. As we show more explicitly in footnote 15, this feature is important to ensure that the incentives for investment do not vanish when we consider short-lived players as in the next section and $\Delta \to 0$. (When we return to long-lived, forward-looking players, incentives for investment will not vanish and similar results apply as $\Delta \to 0$ even if (3) is multiplied with the period of length Δ; see footnote 13).

The next two assumptions are imposed throughout.

Assumption 2 1. c_x and c_s are continuously differentiable, strictly increasing and weakly convex over $\mathbb{R}_+$, and satisfy $\lim_{x\to\infty} c_x'(x) = \infty$ and $\lim_{x\to\infty} c_s'(s) = \infty$.

2.
$$c_s'(\delta) \neq c_x'(\delta).$$

3.
$$\frac{|c_s''(\delta) - c_x''(\delta)|}{\min\{c_x''(\delta), c_s''(\delta)\}} < \frac{1}{\sup_z |h'(z)|}.$$

4.
$$c_s'(0) + \gamma_s \geq c_x'(\delta) \text{ and } c_x'(0) + \gamma_x > c_s'(\delta).$$

Part 1 of Assumption 2 is standard. Part 2 is imposed for simplicity and rules out the non-generic case where the marginal cost of investment at δ is exactly equal for the two parties. Part 3 is also imposed for technical convenience, and is quite weak. For example, if the gap between $c_x''(\delta)$ and $c_s''(\delta)$ is small, this condition is automatically satisfied. We will flag its role when we come to our analysis, but anticipating that discussion, it makes it much easier for us to establish the instability of some "uninteresting" steady-state equilibria. Part 4 ensures that the marginal cost of each player in the increasing returns region (when $x < \gamma_x$ or $s < \gamma_s$) when making zero investment is greater than the marginal cost of the other player outside this region when evaluated at δ — the marginal cost on the right-hand side is evaluated at δ since, as our above transformation showed, the level of investment necessary for maintaining any positive steady-state level of capacity is δ. We will flag the role of this assumption when we come to our formal analysis.

Assumption 3 1. h exists everywhere, and is differentiable, single-peaked and symmetric around zero.

2. For each $z \in \{x, s\}$,
$$c_z'(0) > h(1).$$

3. For each $z \in \{x, s\}$,
$$\min\{h(0) - \gamma_z; h(\gamma_z)\} > c_z'(\delta).$$

Part 1 contains the second key assumption for our analysis — single peakedness and symmetry of h around 0 (differentiability is standard). This assumption not only simplifies our analysis as it ensures that $h(x - s) = h(s - x)$ and $h'(x - s) = -h'(s - x)$, but also implies that incentives for investment are strongest when x and s are close to each other. We highlight the role of this feature below as well.[14]

[14] The result that incentives for investment are strongest when the two sides are evenly matched is more general than the specification used here. For example, suppose that we have a contest function where the probability that the state wins is $\frac{k(s)}{k(s)+k(x)+\eta}$ and the probability that society wins is $\frac{k(x)}{k(s)+k(x)+\eta}$, where $k(\cdot)$ is an increasing, differentiable function, and $\eta \geq 0$ is a constant. In this case, the marginal return to increasing investment for the state is $\frac{k'(s_t)(k(x_t)+\eta)}{(k(s)+k(x)+\eta)^2}$, and the expression for society is also similar. It can be verified that, when $\eta = 0$, the cross-partial derivative of this expression is positive when $s_t > x_t$, and negative when $s_t < x_t$. When $\eta > 0$, the same result holds provided that s_t is sufficiently larger than x_t.

Part 2 imposes that when a player has the maximum gap between itself and the other player, it has no further incentives to invest. Part 3, on the other hand, ensures that at or near the point where conflict capacities are equal, there are sufficient incentives to increase conflict capacity. Both of these assumptions restrict attention to the part of the parameter space of greater interest to us.

2.4 Microfoundations for Economic and Political Decisions

The model presented so far is reduced-form in many dimensions. One of those is the nature of the actions taken by "society". In this subsection, we briefly outline a model of conflict and production, which maps into the reduced-form setup described so far. Suppose that society consists of a state (ruler) and a number of small producers, each with the production function

$$F(g_t, k_{it}),$$

where g_t is a measure of public good provision (such as infrastructure, bureaucratic services or law enforcement) at time t, and k_{it} designates the capital investment of producer i. To simplify notation in this subsection, we suppress time subscripts.

The cost of public good investment depends on the state's "infrastructural power", which is denoted by s_t. We write this cost as $\Gamma_g(g \mid s)$. This dependence captures the fact that investing in public good provision will be much more difficult for the state when it is not otherwise powerful. There is also a separate cost of increasing the infrastructural power of the state as specified in the text. In addition, this infrastructural power of the state will also determine the state's relationship with society.

The producers, on the other hand, individually choose their capital level, but also jointly choose the extent to which they coordinate their political (and perhaps also economic) actions, which we denote by x. A higher degree of coordination among the producers might (but need not) impact their costs of investing in capital, which we write as $\Gamma_k(k \mid x)$, and this dependence might reflect the fact that a greater degree of coordination among the producers enables them to help each other or develop greater trust in production relations or internalize some externalities. More importantly, such coordination impacts how they can deal with the state's demands, and in the context of our model also stands for social norms that society develops for managing political hierarchy as our historical cases also emphasize. We assume that the cost of investing in x is as specified earlier in this section.

Note that the assumptions that only s and x, and not g and k, build on their non-depreciated stock is for simplicity, and facilitate the comparison with our reduced-form model.

The political game takes the following form: first, the state and civil society simultaneously choose their investments, g and k. Then, the state announces a tax rate τ on the output of the producers. If the producers accept this tax rate, it is collected and the remainder is kept by the producers. If they refuse to recognize this tax rate, there will be a conflict between state and society, the outcome of which will be determined by s and x in a manner similar to the conflict in the text. In particular, the state will win this conflict if (4) above holds, and if so, it can extract the entire output of producers, while if the inequality is reversed, society wins, and the state will not be able to collect any taxes.

The equilibrium can be solved by backward induction within the period, starting from the tax decision of the state. Given the conflict technology we have just specified, it is clear that if the tax rate τ is greater than the likelihood of the state winning the conflict, $H(s-x)$, then there will be a conflict. We may thus focus, without loss of any generality, on the case in which $\tau = H(s-x)$. Then the state's maximization problem can be written as

$$H(s-x)F(g,k) - \Gamma_g(g \mid s) - \tilde{C}_s(s, s_{-\Delta}),$$

where $\tilde{C}_s$ is a cost function for the power of the state similar to the one specified in the text, $s_{-\Delta}$ denotes last period's state strength, and k is the common physical capital investment level of all agents. The solution to this problem for g can be summarized as

$$g = g^*(x, k, s).$$

Note that even though $s_{-\Delta}$ influences s, it does not directly impact the choice of g.

Similarly, recalling that $1 - H(s-x) = H(x-s)$, the maximization problem of citizens can be written as

$$H(x-s)F(g,k) - \Gamma_k(k \mid x) - \tilde{C}_x(x, x_{-\Delta}),$$

with solution

$$k = k^*(x, g, s).$$

Solving this equation together with the equation for g, we can eliminate dependence on the economic decision of the other party, and obtain an equilibrium, expressed as $g = g^{**}(x,s)$, and $k = k^{**}(x,s)$. Substituting these into the payoff functions, we obtain a simplified maximization problem for both players similar to the one described above. In particular, the relevant equations become:

$$H(s-x)f(x,s) - C_s(s, s_{-\Delta} \mid x),$$

and

$$H(x-s)f(x,s) - C_x(x, x_{-\Delta} \mid s),$$

where

$$f(x,s) = F(g^{**}(x,s), k^{**}(x,s)),$$

$$C_s(s, s_{-\Delta} \mid x) = \Gamma_g(g^{**}(x,s) \mid s) + \tilde{C}_s(s, s_{-\Delta})$$

and

$$C_x(x, x_{-\Delta} \mid s) = \Gamma_k(k^{**}(x,s) \mid s) + \tilde{C}_x(x, x_{-\Delta}).$$

The only complication relative to the model presented so far is that because the cost functions depend on the equilibrium action choices of the other player, there may be non-uniqueness issues, and thus the relevant statements now will have to be conditional on a particular equilibrium selection.

In the rest of our analysis, we focus on the reduced-form model presented above, even if the microfoundation presented here are useful for interpreting the economic decisions in this model.

3 Equilibrium with Short-Lived Players

We now present our main results about the dynamics of the power of state and civil society, focusing on the non-overlapping generations setup, where at each point in time, each side of the conflict is represented by a single short-lived agent who will be replaced by a new agent from the same side next period. This ensures that when players take decisions today they do not internalize their impact on the future evolution of the power of either party.

3.1 Preliminaries

Suppose that the above-described society is populated by non-overlapping generations of agents — on the one side representing the elite (state) and on the other, civil society.

With these assumptions, at each time t, civil society maximizes

$$H(x_t - s_t) - \Delta \cdot C_x(x_t, x_{t-\Delta})$$

by choosing x_t (or equivalently i_t^x), taking $x_{t-\Delta}$ as given. Simultaneously, the elite maximize

$$H(s_t - x_t) - \Delta \cdot C_s(s_t, s_{t-\Delta})$$

by choosing s_t, taking $s_{t-\Delta}$ as given. A dynamic (Nash) equilibrium with short-lived players is given by a sequence $\{x_{k\Delta}^*, s_{k\Delta}^*\}_{k=0}^{\infty}$ such that $x_{k\Delta}^*$ is a best response to $s_{k\Delta}^*$ given $x_{(k-1)\Delta}^*$, and likewise $s_{k\Delta}^*$ is a best response to $x_{k\Delta}^*$ given $s_{(k-1)\Delta}^*$.

The investment decisions of both elites and civil society are then determined by their respective first-order conditions (with complementary slackness). In particular, at time t, we have:[15]

$$\begin{array}{ll} h(x_t - s_t) \leq c_x'(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_x - x_{t-\Delta}\} & \text{if } \frac{x_t - x_{t-\Delta}}{\Delta} = -\delta \text{ or } x_t = 0, \\ h(x_t - s_t) \geq c_x'(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_x - x_{t-\Delta}\} & \text{if } x_t = 1, \\ h(x_t - s_t) = c_x'(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_x - x_{t-\Delta}\} & \text{otherwise,} \end{array}$$

and

$$\begin{array}{ll} h(s_t - x_t) \leq c_s'(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_s - s_{t-\Delta}\} & \text{if } \frac{s_t - s_{t-\Delta}}{\Delta} = -\delta \text{ or } s_t = 0, \\ h(s_t - x_t) \geq c_s'(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_s - s_{t-\Delta}\} & \text{if } s_t = 1, \\ h(s_t - x_t) = c_s'(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_s - s_{t-\Delta}\} & \text{otherwise.} \end{array}$$

The first line of either expression applies when the relevant player has chosen zero investment so that its state variable shrinks as fast as it can (at the rate δ), or is already at its lower bound $x_t = 0$ or $s_t = 0$. In this case, we have the additional cost of investment on the right-hand side, and also the optimality condition is given by a weak inequality, since at this lower boundary, the marginal benefit could be strictly less than the marginal cost of investment. The second line, on the other hand, applies when the state variable takes its maximum value, 1, and in this case the marginal benefit could be strictly greater than the marginal cost of investment. Away from these boundaries, the third line applies and requires that the marginal benefit equal the marginal cost. Note also that the marginal

[15] Following up on footnote 13, we can more clearly see the role that Δ in front of the cost function plays here: without this term (or equivalently if the return was also multiplied by Δ), the marginal cost of investment would be multiplied by $1/\Delta$, and thus as $\Delta \to 0$, investments would converge to zero. This is because short-lived players that are not forward-looking do not take the impact of their instantaneous investments on future stocks (and have infinitesimal impact on the current stock).

benefit for civil society is the same as the marginal benefit for the state — since $h(s_t - x_t) = h(x_t - s_t)$. On the other hand, we also have from Assumption 3 that changes in the marginal benefits of the two players are the converses of each other — that is, $h'(s_t - x_t) = -h'(x_t - s_t)$.

Now letting $\Delta \to 0$, we obtain the following continuous-time first-order optimality (and thus equilibrium) conditions

$$\begin{array}{lc} h(x_t - s_t) \leq c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } \dot{x}_t = -\delta \text{ or } x_t = 0, \\ h(x_t - s_t) \geq c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } x_t = 1, \\ h(x_t - s_t) = c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{otherwise}, \end{array} \tag{5}$$

and

$$\begin{array}{lc} h(s_t - x_t) \leq c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{if } \dot{s}_t = -\delta \text{ or } s_t = 0, \\ h(s_t - x_t) \geq c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{if } s_t = 1, \\ h(s_t - x_t) = c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{otherwise}. \end{array} \tag{6}$$

In what follows, we work directly with these continuous-time first-order optimality conditions. Moreover, it is straightforward to see that in continuous time, away from the boundaries of $[0, 1]^2$ these first-order optimality conditions will hold as equality, and thus the dynamics of state and civil society strength can be represented by the following two differential equations:

$$\begin{aligned} \dot{x} &= \max\{(c_x')^{-1}(h(x - s) - \max\{\gamma_x - x, 0\}); 0\} - \delta \\ \dot{s} &= \max\{(c_s')^{-1}(h(s - x) - \max\{\gamma_s - s, 0\}); 0\} - \delta. \end{aligned} \tag{7}$$

The roles of the two key assumptions highlighted above — the single-peakedness of h and the increasing returns aspect of the cost function — are evident from (7). First, when x and s are close to each other, $h(x - s)$ is large, and thus both of these variables will tend to grow further. Conversely, when x and s are far apart, $h(x - s)$ is small, and investment by both parties is discouraged. This observation captures the key economic force that will lead to the emergence of different dynamics of state-society relations and different types of states in our setup (in the Appendix, we see that this same property holds with other formulations of the contest function). Secondly, the presence of the max term implies that once the conflict capacity of a party falls below a critical threshold (γ_x or γ_s), there is an additional force pushing towards further reduction in this capacity.

3.2 Dynamics of the Strength of Civil Society and the State

Our main result in this section is summarized in the next proposition.

Proposition 1 *Suppose Assumptions 1, 2 and 3 hold. Then there are three (locally) asymptotically stable steady states:*

1. $x^* = s^* = 1$.
2. $x^* = 0$ *and* $s^* \in (\gamma_s, 1)$.
3. $x^* \in (\gamma_x, 1)$ *and* $s^* = 0$.

This proposition shows that there exist three relevant (asymptotically stable) steady states, one corresponding to an inclusive state, one corresponding to a despotic state and one to a weak state.

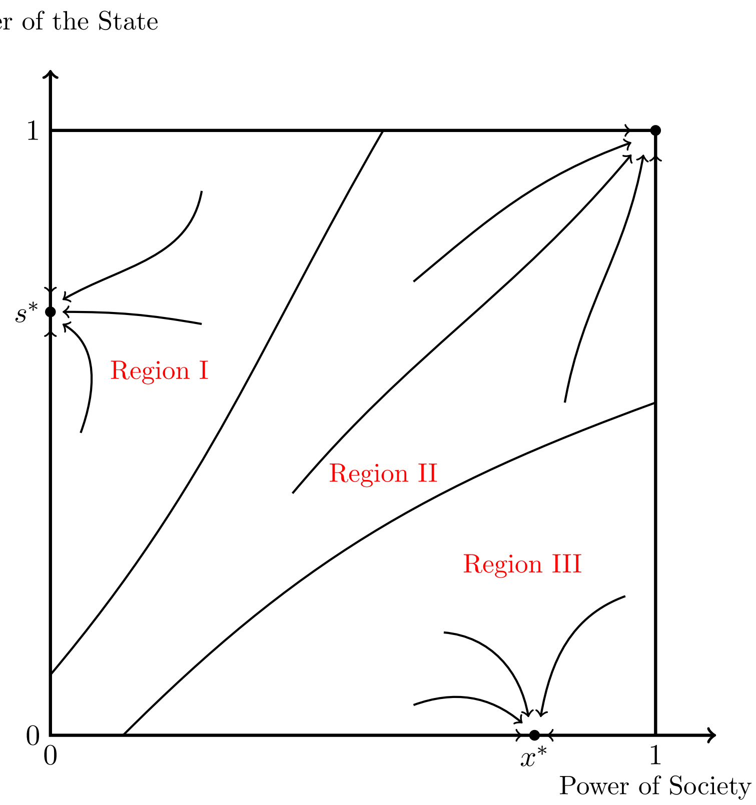

Figure 2: Steady states and their local dynamics.

The intuition, as already anticipated, is that when we are in the neighborhood of the steady state $x^* = s^* = 1$, $h(x - s)$ is large, encouraging both parties to move further towards $x^* = s^* = 1$. In contrast, in the neighborhood of $x^* = 0$ or $s^* = 0$, $h(x - s)$ is small, and neither party has as strong incentives to invest, and in fact, one of them ends up with zero conflict capacity.[16]

The steady states presented in Proposition 1 and their local dynamics are depicted in Figure 2. Our analysis so far establishes the dynamics in the neighborhoods of these steady states. After providing the proof of Proposition 1, we turn to a characterization of global dynamics.

3.3 Proof of Proposition 1

We start with a series of lemmas on the steady-state equilibria of this model, and their stability properties. Before presenting these results, we remark that, mathematically, there can be three types of steady states: (i) those in which the party in question (say the civil society) chooses a positive level conflict capacity, and thus we will have $x_t^* = x^* \in (0, 1)$, so that the marginal cost of investment is simply $c_x'(\delta) + \max\{\gamma_x - x^*, 0\}$, which is equal to the benefit from this conflict capacity; (ii) those in which we have zero conflict capacity, in which case the marginal cost of investment, $c_x'(0) + \gamma_x$, is greater than or equal to the benefit from building further conflict capacity; (iii) those in which the party in question has conflict capacity equal to 1, in which case marginal cost of investment, $c_x'(\delta)$, is

[16]Under Assumption 1, there is no social benefit in reaching the steady state $x^* = s^* = 1$, since the capacities of the state and society do not contribute to the size of the social surplus. This will be relaxed below when we consider the general environment in which x and s contribute to the size of total surplus.

less than or equal to the benefit from building additional conflict capacity.

Lemma 1 *There exists a (locally) asymptotically stable steady state with* $x^* = s^* = 1$.

The proof of this lemma and all of our subsequent results are provided in the Appendix. It shows that under our maintained assumptions, both parties investing at their maximum conflict capacity is a steady-state equilibrium. Intuitively, this proposition exploits the fact that when the two players are "neck and neck," they both have strong incentives to invest. If instead we had, say, x much larger than s, then from part 1 of Assumption 3, both $h(x-s)$ and $h(s-x)$ would be smaller than $h(0)$, reducing the investment incentives of both parties. The stronger investment incentives around $x^* = s^* = 1$ are key for maintaining this combination as an (asymptotically stable) steady state — combined with part 2 of Assumption 3, which ensures that these strong incentives are sufficient to guarantee a corner solution. If the inequality in part 2 of Assumption 3 did not hold, $x^* = s^* = 1$ could not be a steady-state equilibrium, and in this case, the only possible steady-state equilibria would be those identified in Lemma 2 below.

The local stability of this steady state is then established by constructing a Lyapunov function. The use of this method is necessitated by the fact that $x^* = s^* = 1$ is at the corner of the feasible set, $[0,1]^2$, and thus dynamics around it cannot be characterized by using linearization methods.

Our next result identifies two additional locally asymptotically stable steady states.

Lemma 2 *There exist two additional (locally) asymptotically stable steady states:*

1. *one with* $x^* = 0$ *and* $s^* \in (\gamma_s, 1)$*, and*

2. *one with* $s^* = 0$ *and* $x^* \in (\gamma_x, 1)$*.*

These two additional steady states have a very different flavor than the steady state in Lemma 1. Now both parties have a lower level of conflict capacity, and one of them is in fact at zero. The intuition is again related to the incentives for investment in conflict capacity: when one party is at zero capacity, $h(\cdot)$ is small for both players, which encourages the first player to build a state with low capacity, and discourages the other player from building further capacity.

Assumptions 2 and 3 play an important role in this lemma as well. Without the boundary conditions in Assumption 3, there could be other steady states with some of them including investments below γ_x and γ_s. Though these steady states would be locally unstable (with the same argument as in Lemma 4 below), it would also become harder to ensure that there exists a locally stable steady state, making us prefer these assumptions.

The next lemma rules out several types of steady states.

Lemma 3 *There is no steady state with (i)* $x^* = s^* = 0$*; or (ii)* $x^* = 0$ *and* $s^* \in (0, \gamma_s)$*, or* $s^* = 0$ *and* $x^* \in (0, \gamma_x)$*; or (iii)* $x^* \in (\gamma_x, 1)$ *and* $s^* \in (\gamma_s, 1)$*.*

There are other types of steady states that could exist, but the next lemma shows that when they do, they will all be asymptotically unstable.

Lemma 4 *All other (possible) steady states are asymptotically unstable.*

Proposition 1 then follows straightforwardly by combining these lemmas.

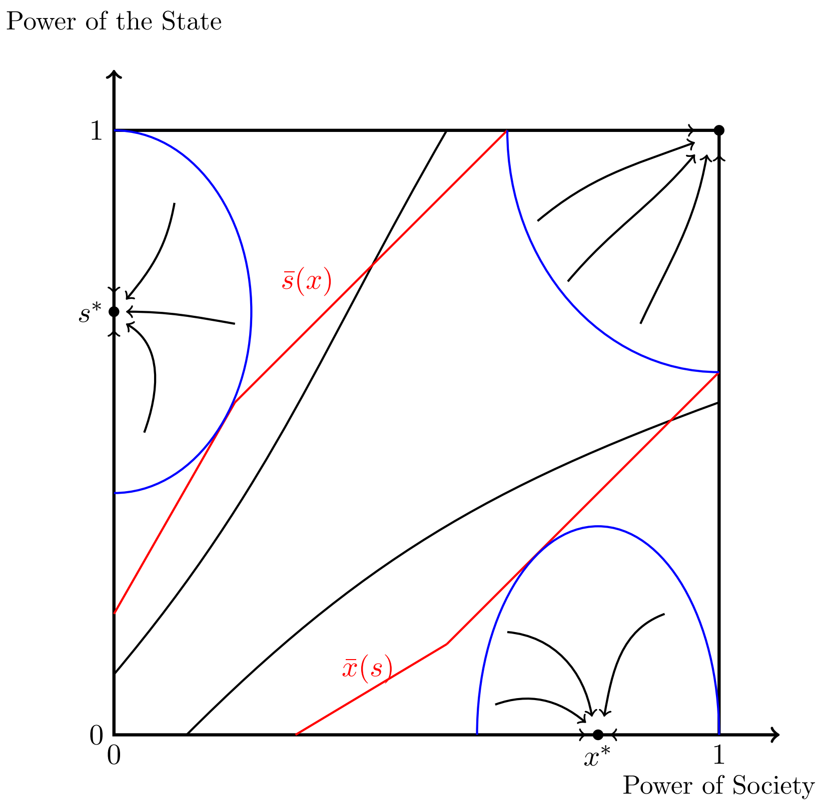

Figure 3: Global dynamics.

3.4 Global Dynamics

We next partially characterize the global dynamics. In particular, we will determine three regions, as shown in Figure 3, separating the phase diagram into basins of attraction of the three asymptotically stable steady states characterized in the previous subsection. For example, starting from Region I, equilibrium dynamics converge to the steady state with $x^* = 0$ and $s^* \in (\gamma_s, 1)$; from Region II, convergence is to the steady state with $x^* = s^* = 1$; and from Region III, convergence will be to the steady state with $x^* \in (\gamma_x, 1)$ and $s^* = 0$. Unfortunately, it is not possible to determine the boundaries of these regions analytically, but we will be able to characterize subsets thereof explicitly.

Consider first Region II, which is the basin of attraction of the steady state $x^* = s^* = 1$. Recall that the dynamical system for the behavior of the conflict capacity of civil society and state take the form given in (7) above. We proceed by first noting that any subset S of $[0,1]^2$ for which there exists a Lyapunov function $L(x,s)$ such that (i) $S = \{(x,s) : L(x,s) \leq K\}$ for some $K > 0$; (ii) $L(x,s) \geq 0$ for all $(x,s) \in S$, with equality only if $x = s = 1$; and (iii) $\partial L(x,s)/\partial t \leq 0$ for all $(x,s) \in S$, with equality only if $x = s = 1$, is part of the basin of attraction of this steady state.

Let us first construct a subset of the parameters (x,s) such that $\dot{x} \geq 0$ and $\dot{s} \geq 0$, with one of them holding as strict inequality. Let us define $\bar{x}$ such that $c_x'(\delta) = h(\bar{x} - 1)$. Clearly, from Assumption 2 $c_s'(\delta) < h(1 - \bar{x})$. This defines $\mathcal{R}_{II}''= \{(x,s) : x \geq \max\{\gamma_x, \bar{x}\}$ and $s \geq \max\{\gamma_s, \bar{x}\}\}$. This region can be further extended by noting that any combination of (x,s) such that $(c_x')^{-1}(h(x - s) - \max\{\gamma_x - x, 0\}) - \delta \geq 0$ and $(c_s')^{-1}(h(s - x) - \max\{\gamma_s - s, 0\}) - \delta \geq 0$ also satisfies $\dot{x} \geq 0$ and

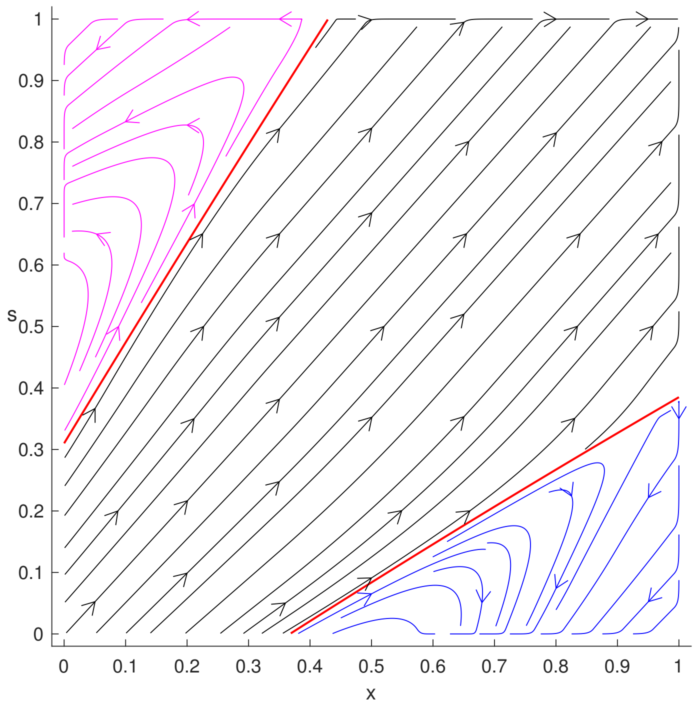

Figure 4: The direction of change of the power of state and society in a simulated example.

$\dot{s} \geq 0$. Let us define $\bar{s}(x)$ such that $h(\bar{s}(x) - x) - \max\{\gamma_s - \bar{s}(x), 0\} - c_s'(\delta) = 0$. Similarly, define $\bar{x}(s)$ such that $h(\bar{x}(s) - s) - \max\{\gamma_x - x, 0\} - c_x'(\delta) = 0$. Both $\bar{s}(x)$ and $\bar{x}(s)$ are upward sloping, and in fact correspond to lines with slope 1 when $s \geq \gamma_s$ and $x \geq \gamma_x$, respectively. Then starting within $\mathcal{R}'_{II} = \{(x, s) : s \leq \bar{s}(x) \text{ and } x \leq \bar{x}(s)\}$, we also have $\dot{x} \geq 0$ and $\dot{s} \geq 0$ (and in fact, $\mathcal{R}''_{II} \subset \mathcal{R}'_{II}$). This region, as well as $\mathcal{R}''_{II}$, is depicted in Figure 3. The shape of the region is intuitive.

Now consider the family of functions, $L(x, s \mid l_x, l_s) = \frac{l_x}{2}(1-x)^2 + \frac{l_s}{2}(1-s)^2$, indexed by $l_x > 0$ and $l_s > 0$. Clearly, for any member of this family, we have that for all $(x, s) \in \mathcal{R}'_{II} \backslash (1, 1)$,

$$\frac{\partial L(x, s \mid l_x, l_s)}{\partial t} = l_x(1-x)\dot{x} - l_s(1-s)\dot{s} < 0.$$

So if we in addition define the subset $\mathcal{R}_{II}$ of $\mathcal{R}'_{II}$ where $L(x, s \mid l_x, l_s) \leq K$, then $\mathcal{R}_{II}$ satisfies the above conditions and by construction is part of the basin of attraction of the steady state $(1, 1)$.

Now consider the problem of choosing K, l_x and l_s such that we achieve the largest set $\mathcal{R}_{II} = \{(x, s) : L(x, s \mid l_x, l_s) \leq K\}$ contained in $\mathcal{R}'_{II}$. Mathematically, let $\mathcal{A}(\mathcal{R}_{II})$ be the area of set $\mathcal{R}_{II}$. Then the problem is to choose

$$\max_{K, l_x, l_s > 0} \mathcal{A}(\mathcal{R}_{II}).$$

Figure 3 shows the construction of region $\mathcal{R}_{II}$ in this manner, which is by construction part of the basin of attraction of the steady state $(1, 1)$.

Subsets of the basins of attraction of the other steady states can be constructed analogously and are shown in Figure 3.

We also verify numerically that dynamics take the form shown in Figures 2 and 3. In Figure 4, we depict the vector field for a specific parameterization of the model. We take $f(x, s) = 0.6$, and choose H to be a raised cosine distribution over $[-1,1]$ with mean $\mu = 0$, which is single-peaked and symmetric consistent with Assumption 3.[17] The cost functions of the state and civil society are

$$c_x(i) = 3.25 \times i^2 \text{ (for } i \in [0,10]) \text{ and } c_s(i) = 3.25 \times i^2 \quad \text{(for } i \in [0,15]),$$

and outside of these ranges, the cost functions become vertical, placing a bound on investment levels.[18] In addition, we set $\gamma_x = 0.35$, $\gamma_s = 0.35$, and $\delta = 0.1$. The figure verifies the qualitative characterization provided so far.

3.5 The "Conditional" Effects of Changes in Initial Conditions

Though we will discuss comparative statics (or "comparative dynamics") in greater detail in Section 5, here we undertake a simple exercise: change the initial conditions and trace the effects of these on equilibrium dynamics. The immediate but important conclusion is that the same change in initial conditions, starting from different parts of the state space, can have drastically different implications.

Consider an increase in s_0 to $s_0 + \bar{s}$. This can leave us in the same region as before, in which case the equilibrium trajectory will be shifted uniformly up, but the long-run outcome will remain unchanged. Alternatively, this increase can shift us from, say, Region III to Region II, in which case not only the equilibrium trajectory but also the long-run outcome will change, and in fact it will involve greater state capacity. However, depending on the exact value of (x_0, s_0), an increase of the same amount $\bar{s}$ could also shift us from Region II to Region I, in which case the impact on the long-run state capacity will be negative instead of positive. This illustrates, while also providing a simple proof, that the effects of changes in initial conditions in this model are *conditional* — they depend exactly on where we start. This discussion thus establishes:

Proposition 2 *The effects of changes in the initial conditions* (x_0, s_0) *on equilibrium dynamics and the long-run outcome of the society are conditional in the sense that these depend on which region we move out of and into.*

4 Equilibrium with Forward-Looking Players

In this section, we analyze our general framework with long-lived, forward-looking players. After briefly describing preferences, we first show that for high rates of discounting, equilibrium behavior converges to behavior with short-lived players, which we characterized in the previous section. We then numerically study the dynamics of the equilibrium for a range of discount rates. In this section, we continue to impose Assumption 1, and then relax it in the next section.

4.1 Preferences

We start with the discrete time model. The technology of investment and conflict are the same as in the previous two sections. The only difference is that now both civil society and state are long-lived

[17] Assumption 1 imposed that $f(x,s) = 1$ rather than setting it equal to a constant, say ϕ_0, in order to reduce the number of parameters. We consider a more general surplus function in Assumption 1$'$ below. Setting $\phi_0 = 0.6$ enables us to construct an example with more equally-sized regions.

[18] This bound plays no role in the numerical results reported here, but facilitates convergence when we consider the dynamic model with the same parameterization and low discount rates in the next section.

and forward-looking. To maximize the parallel with the model with short-lived players, we assume that both players again correspond to sequences of non-overlapping generations, but each generation has an exponentially-distributed lifetime or equivalently, a Poisson end date with parameter, $1-\beta$, where $\beta = e^{-\rho\Delta}$. We assume that this random end date is the only source of discounting. Clearly, this specification guarantees that as the period length Δ shrinks, discounting between periods will also decline (and the discount factor will approach 1). Again to maximize the parallel with our static model, we assume that in expectation, there is one instance of conflict between the two players during the lifetime of each generation. Since with this Poisson specification, the expected lifetime of his generation is $1/(1-\beta)$, this implies that a conflict arrives at the rate $1-\beta$.[19]

4.2 Main Result

The main result we prove in this forward-looking model is provided in the next proposition.

Proposition 3 *Suppose Assumptions 1, 2 and 3 hold. Then there exist discount rates $\bar{\rho} \geq \underline{\rho} > 0$ such that for all $\rho > \bar{\rho}$, there are three (locally) asymptotically stable steady states:*

1. $x^* = s^* = 1$.
2. $x^* = 0$ *and* $s^* \in (\gamma_s, 1)$.
3. $x^* \in (\gamma_x, 1)$ *and* $s^* = 0$.

Moreover, for all $\rho < \underline{\rho}$, there exists a unique globally stable steady state $x^* = s^* = 1$.

This result thus shows that the main insights from our analysis apply provided that players, though forward-looking, are sufficiently impatient. We note that this result is not a simple consequence of the fact that as we consider larger and larger values of ρ, players are becoming closer to myopic. It necessitates establishing properties of the the relevant value functions and their derivatives in the limit.[20]

4.3 Proof of Proposition 3

With the specification introduced above, we can straightforwardly represent the maximization problem of each player as a solution to a recursive, dynamic programming problem, written as

$$V_x(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta) = \max_{x_t \in [0,1]} \left\{ (1-\beta) H(x_t - s_t) - \Delta \cdot C_x(x_t, x_{t-\Delta}) + \beta V_x(x_t, s'^*(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta), \beta; \Delta) \right\}, \tag{8}$$

[19] An alternative specification of the model with long-lived players which leads to identical equations, but eschews the parallel with the static model, is to assume that both players are infinitely lived and discount the future at the rate $\beta = e^{-\rho\Delta}$ and there is a conflict during each interval of length Δ. Recall from footnote 15 that in this case there will be no investment when $\Delta \to 0$ with short-lived players (because they do not take into account the benefit from increasing future conflict capabilities), but incentives for investment do not disappear with long-lived, forward-looking players even as $\Delta \to 0$ (because they *do* take into account the benefit from increasing future conflict capabilities).

[20] When ρ is between $\underline{\rho}$ and $\bar{\rho}$, we may have a situation in which one of the two corner steady states disappears while the other one still exists.

and

$$V_s(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta) = \max_{s_t \in [0,1]} \left\{ (1-\beta) H(s_t - x_t) - \Delta \cdot C_s(s_t, s_{t-\Delta}) + \beta V_s(x'^*(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta), s_t, \beta; \Delta) \right\}. \tag{9}$$

Several things are important to note. First, as anticipated in the previous section, we multiply the flow costs with Δ, but not the benefits, since these capture life-time benefits from conflict, and we have conditioned on Δ in writing the value functions for emphasis. Second, notice that we have already imposed the boundary conditions, $x_t \in [0,1]$ and $s_t \in [0,1]$, in the maximization problems. Third, $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ are the policy functions, which give the next period's values of the state variables as a function of this period's values (and are explicitly conditioned on $\Delta > 0$).

A dynamic equilibrium in this setup as given by a pair of policy functions, $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ which give the next period's values of the state variables as a function of this period's values (for $\Delta > 0$), and each solves the corresponding value function taking the policy function of the other party is given. Once these policy functions are determined, the dynamics of civil society and state strength can be obtained by iterating over these functions.

Since these are standard Bellman equations, the following result is immediate (throughout this proof we take $(x,\ s,\ \beta) \in [0,1]^3$).

Lemma 5 *For any $\Delta > 0$, $V_x(x, s, \beta; \Delta)$ and $V_s(x, s, \beta; \Delta)$ exist and are continuously differentiable in x, s and Δ. Moreover, $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ and are continuous in x, s and Δ.*

In particular, from (8) and (9), as $\beta \to 0$, $V_x(x, s, \beta; \Delta) \to V_x(x, s, \beta = 0; \Delta)$ and $V_s(x, s, \beta; \Delta) \to V_s(x, s, \beta = 0; \Delta)$. But since $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ are maximizers of the continuous (and bounded) functions, (8) and (9), we can apply Berge's maximum theorem to conclude that $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ are also continuous, particularly in β, and thus $x'^*(x, s, \beta; \Delta) \to x'^*(x, s, \beta = 0; \Delta)$ and $s'^*(x, s, \beta; \Delta) \to s'^*(x, s, \beta = 0; \Delta)$, and thus for β sufficiently close to 0, we have that $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ are approximately the same as their myopic values. Therefore, there exists $\bar{\beta} > 0$, such that for all $\beta < \bar{\beta}$, a steady state of the dynamical system given by $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ exists and is locally stable if and only if it is a locally stable steady state of the myopic model.

This argument establishes that the forward-looking, discrete-time dynamics when the discount factor is sufficiently close to 0 will have the same locally stable steady states as the myopic, discrete-time dynamics. In the previous section, we approximated the discrete-time dynamics with their continuous-time limit, and it is also convenient to do the same here, and to maximize the parallel, this is how we have stated the proposition.

We can also observe that when the discount factor $\beta \to 1$, the two steady states other than $(1,1)$ disappear. The argument is simple: take the steady state with $x = 0$, where the society's flow return is zero. If civil society invests at a high level for a finite number of periods, this will ensure that $x \geq \gamma_x$, eliminating the region of higher costs of investment for civil society, and thus taking x to 1 (which gives the society a positive flow return). When β is arbitrarily close to 1, the costs of investing at a high level for a finite number of periods are negligible, and hence such a deviation is profitable for civil society. This argument, again from continuity, ensures that there exists $\underline{\beta}^x < 1$ such that for $\beta > \underline{\beta}^x$, $x = 0$ is not consistent with a steady state. With the parallel argument, we also have that there exists $\underline{\beta}^s < 1$ such that for $\beta > \underline{\beta}^s$, $s = 0$ cannot be part of a steady state. Then, for $\beta > \underline{\beta} = \max\{\underline{\beta}^x, \underline{\beta}^s\}$ only $(1, 1)$ remains as an asymptotically stable steady state.

The next subsection discusses the continuous-time limit and also derives the continuous-time Hamilton-Jacobi-Bellman (HJB) equations, which can be used to characterize the equilibrium more generally. We then come back to completing the proof of Proposition 3.

4.4 Continuous-Time Approximation

For characterizing the equilibrium for any value of the players' impatience, we can once again use the continuous-time approximation by taking the limit $\Delta \to 0$, which shrinks the period length (and correspondingly adjusts the discount factor $\beta = e^{-\rho\Delta}$, so that the discount rate remains constant at ρ). In this limit, conditions on β translate into conditions on ρ. More specifically, we have:

Lemma 6 *As $\Delta \to 0$, the value functions $V_x(x, s, \beta; \Delta)$ and $V_s(x, s, \beta; \Delta)$ converge to their continuous-time limits $V_x(x, s)$ and $V_s(x, s)$, and the policy functions $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ converge to their continuous-time limits $x'^*(x, s)$ and $s'^*(x, s)$.*[21]

The continuous-time Hamilton-Jacobi-Bellman (HJB) equations can be obtained as follows. First rearrange (8) evaluated at the optimal choices and divide both sides by Δ to obtain

$$\begin{aligned} & \frac{1-\beta}{\Delta} V_x(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta) \\ = \; & \max_{x_t \geq 0} \left[\frac{1-\beta}{\Delta} H(x_t - s_t) - C_x\left(x_t, x_{t-\Delta}\right) + \beta \frac{V_x(x_t, s^*_\Delta(x_{t-\Delta}, s_{t-\Delta}), \beta; \Delta) - V_x(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta)}{\Delta} \right]. \end{aligned}$$

Now note that as $\Delta \to 0$, $(1 - \beta) \to 0$ and $(1 - \beta)/\Delta \to \rho$. Moreover the last term in the previous expression tends to the total derivative of the value function with respect to time. Therefore, the continuous-time HJB equation for civil society is

$$\rho V_x(x, s) = \rho H(x - s) + \max_{\dot{x} \geq -\delta} \left\{ -C_x(x, \dot{x}) + \frac{\partial V_x(x, s)}{\partial x} \dot{x} \right\} + \frac{\partial V_x(x, s)}{\partial s} \dot{s}^*(x, s),$$

where we have used the notation $C_x(x, \dot{x})$ to denote the continuous-time cost function as a function of the change in the conflict capacity of civil society, while $\dot{x}^*(x, s)$ and $\dot{s}^*(x, s)$ designate the continuous-time policy functions, conveniently written in terms of the time derivative of the conflict capacities of the two parties. We have also imposed that $\dot{x}$ cannot be less than $-\delta$.

Applying the same argument to (9) and denoting the continues-time cost function for the state by $C_s(s, \dot{s})$, we also obtain

$$\rho V_s(x, s) = \rho H(s - x) + \max_{\dot{s} \geq -\delta} \left\{ -C_s(s, \dot{s}) + \frac{\partial V_s(x, s)}{\partial s} \dot{s} \right\} + \frac{\partial V_s(x, s)}{\partial x} \dot{x}^*(x, s),$$

[21] We also drop the conditioning on the discrete-time discount factor β in writing the continuous-time value and policy functions and do not add conditioning on its continuous-time equivalent, the discount rate ρ to simplify the notation.

The first-order optimality conditions for civil society are given by

$$\begin{aligned}
\frac{\partial C_x(x,\dot{x})}{\partial \dot{x}} &= \frac{\partial V_x(x,s)}{\partial x} && \text{if} \quad -\delta < \dot{x}(x,s), \text{ and } x \in (0,1), \\
\frac{\partial C_x(x,\dot{x})}{\partial \dot{x}} &\leq \frac{\partial V_x(x,s)}{\partial x} && \text{if} \quad x = 1, \\
\frac{\partial C_x(x,\dot{x})}{\partial \dot{x}} &\geq \frac{\partial V_x(x,s)}{\partial x} && \text{if} \quad \dot{x}(x,s) = -\delta \text{ or } x = 0.
\end{aligned} \tag{10}$$

In the first case, when we have an interior solution, we can also write

$$\dot{x} = \begin{cases} (c'_x)^{-1}\left(\frac{\partial V_x(x,s)}{\partial x} - \gamma_x + x\right) & \text{if } x \leq \gamma_x \\ (c'_x)^{-1}\left(\frac{\partial V_x(x,s)}{\partial x}\right) & \text{if } x > \gamma_x \end{cases}. \tag{11}$$

The first-order conditions for the state are also similar, and for an interior solution, they yield

$$\dot{s} = \begin{cases} (c'_s)^{-1}\left(\frac{\partial V_s(x,s)}{\partial s} - \gamma_s + s\right) & \text{if } s \leq \gamma_s \\ (c'_s)^{-1}\left(\frac{\partial V_s(x,s)}{\partial s}\right) & \text{if } s > \gamma_s \end{cases}. \tag{12}$$

4.5 Completing the Proof of Proposition 3

We have already established that, for fixed $\Delta > 0$, as $\beta \to 0$, the solution and the implied dynamics in the forward-looking case converge to their equivalents. This ensures that there exists $\bar{\beta} > 0$, such that for $\beta < \bar{\beta}$, the dynamics of the myopic and forward-looking cases will be sufficiently close to each other and thus will have the same set of locally stable steady states. Then taking the limit $\Delta \to 0$, this can be expressed in terms of the continuous-time approximations for both systems, thus establishing that for $\rho > \bar{\rho}$, the locally stable steady states in Propositions 1 and 3 coincide. Similarly, we also argue that for $\beta > \underline{\beta}$ (where $\underline{\beta} < 1$), there cannot exist a steady state with either $x = 0$ or $s = 0$, and thus the only asymptotically stable steady state is $(1,1)$. With the same argument, when $\Delta \to 0$, we can then conclude that there exists $\underline{\rho} > 0$ such that for discount rates below this, only $(1,1)$ remains as the asymptotically stable steady state.

4.6 Numerical Results

We next provide a numerical characterization of the dynamics in the forward-looking model. We use the same formulation of the cost function and parameter values as above. The critical threshold for ρ is computed as $\bar{\rho} = 100$, and for discount rates above this value, the vector field is identical to that shown in Figure 4, confirming that for high discount rates the equilibrium dynamics of the model with forward-looking agents coincide with the equilibrium of the model with myopic agents as claimed in Proposition 1. In Figure 5, we also show the implied vector field when ρ is smaller than $\bar{\rho}$ (in this instance, $\rho = 30$), which illustrates the very different dynamics in this case.

5 General Characterization

In this section, we relax Assumption 1. Since we have established the equivalence of the myopic and forward-looking models when the discount rate is sufficiently large in the latter (which is a result that does not depend in any way on Assumption 1), here we focus on a model with forward-looking

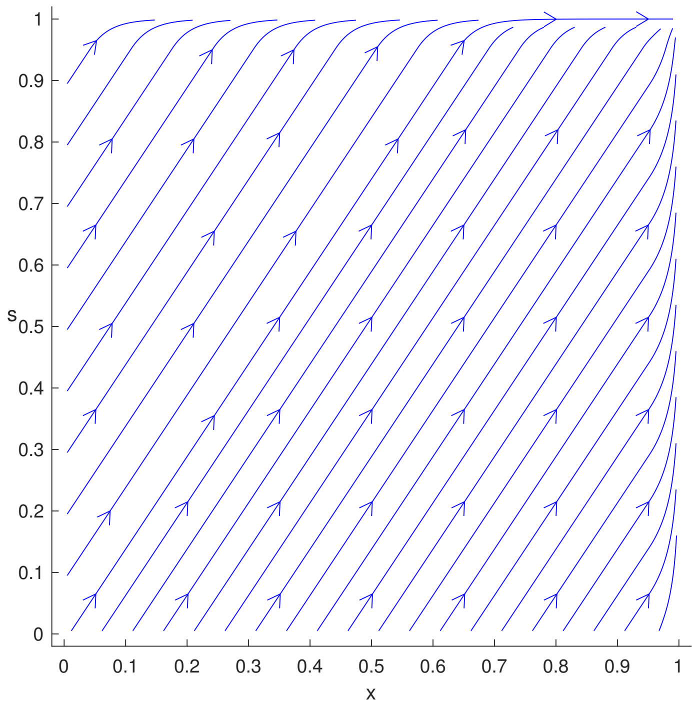

Figure 5: The direction of change of the power of state and society in a simulated example with $\rho = 30$.

players. We also simplify the analysis throughout by assuming that f is linear as specified in the next assumption, which replaces Assumption 1.

5.1 Modified Assumptions

Assumption 1′ $f(x,s) = \phi_0 + \phi_x x + \phi_s s$, where $\phi_0 > 0$, $\phi_x > 0$ and $\phi_s > 0$.

Our other two assumptions also require some minor modifications, which are provided next.

Assumption 2′

1. c_x and c_s are continuously differentiable, strictly increasing and weakly convex, and satisfy $\lim_{x\to\infty} c_x'(x) = \infty$ and $\lim_{x\to\infty} c_s'(s) = \infty$.

2. $$c_s'(\delta) \neq c_x'(\delta).$$

3. $$\frac{|c_s''(\delta) - c_x''(\delta)|}{\min\{c_x''(\delta), c_s''(\delta)\}} < \inf_z \frac{2h(z)(\phi_s + \phi_x)}{|h'(z)|\,(\phi_0 + \phi_s + \phi_x)}.$$

4. $$c_s'(0) + \gamma_s \geq c_x'(\delta) \text{ and } c_x'(0) + \gamma_x > c_s'(\delta).$$

The minor modifications in parts 3 and 4 are in view of the fact that marginal benefits of investment are different between the state and civil society. For same reason, we also modify Assumption 3 as follows.

Assumption 3′ 1. h exists everywhere, and is differentiable, single-peaked and symmetric around zero.

2. For each $z \in \{x, s\}$,

$$c_z'(0) > h(1)(\phi_0 + \phi_z) + H(1)\phi_z.$$

3. For each $z \in \{x, s\}$,

$$\min\{h(0)\phi_0 + H(0)\phi_z - \gamma_z; h(\gamma_z)(\phi_0 + \phi_z\gamma_z) + H(\gamma_z)\phi_z\} > c_z'(\delta).$$

Under these assumptions, the first-order optimality conditions with short-live players (in continuous time) are modified in the following straightforward fashion:

$$\begin{array}{lc} h(x_t - s_t)(\phi_0 + \phi_x x + \phi_s s) + H(x_t - s_t)\phi_x \leq c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } \dot{x}_t = -\delta \text{ or } x_t = 0, \\ h(x_t - s_t)(\phi_0 + \phi_x x + \phi_s s) + H(x_t - s_t)\phi_x \geq c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } x_t = 1, \\ h(x_t - s_t)(\phi_0 + \phi_x x + \phi_s s) + H(x_t - s_t)\phi_x = c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{otherwise.} \end{array}$$

and

$$\begin{array}{lc} h(s_t - x_t)(\phi_0 + \phi_x x + \phi_s s) + H(s_t - x_t)\phi_s \leq c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{if } \dot{s}_t = -\delta \text{ or } s_t = 0, \\ h(s_t - x_t)(\phi_0 + \phi_x x + \phi_s s) + H(s_t - x_t)\phi_s \geq c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{if } s_t = 1, \\ h(s_t - x_t)(\phi_0 + \phi_x x + \phi_s s) + H(s_t - x_t)\phi_s = c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{otherwise,} \end{array}$$

5.2 Main Result

We have the following straightforward result.

Proposition 4 *Suppose that Assumptions 1′, 2′ and 3′ hold. Then Propositions 1 and 3 apply.*

5.3 Comparative Statics

In this subsection, we discuss how changes in parameters affect the steady states and the dynamics of equilibrium. We focus on the effects of changes in the parameters ϕ_x, ϕ_s, γ_x and γ_s as well as the cost functions c_x and c_s. The effects of changes in initial conditions are identical to those already discussed in Section 3.5

Assumption 3′ guarantees that $x^* = 1$ and $s^* = 1$ is a steady state. There are also at least two interior steady states. These steady states are one of two types. The first type is given by $x^* = 0$ and any s^* that satisfies the following equation:

$$h(s)(\phi_0 + \phi_s s) + H(s)\phi_s = c_s'(\delta).$$

The second type is given by $s^* = 0$ and any x^* that satisfies the following equation

$$h(x)(\phi_0 + \phi_x x) + H(x)\phi_x = c_x'(\delta).$$

Assumption 3′ guarantees that at least one steady state of each type exists. We impose the following assumption to make sure that only one steady state of each type exist:

Assumption 4 $h(y)(\phi_0 + \phi_z y) + H(y)\phi_z$ is a decreasing function of $y \geq 0$ for $z \in \{s, x\}$.

This assumption is fairly mild. The following two conditions would be sufficient to guarantee it: (i) ϕ_z is small, in which case the fact that, from Assumption 3′, $h(y)$ is decreasing for $y \geq 0$ ensures that this assumption is also satisfied, or that (ii) the elasticity of the h function is greater than 1/2, in which case for any value of ϕ_0, Assumption 4 is satisfied.

Let us focus on the comparative statics of the steady state with $x^* = 0$ and $s^* \in (\gamma_s, 1)$. The other case is identical. s^* solves the following equation:

$$h(s^*)(\phi_0 + \phi_s s^*) + H(s^*)\phi_s = c_s'(\delta). \tag{13}$$

The parameter ϕ_x does not directly appear in this equation. Therefore,

$$\frac{\partial s^*}{\partial \phi_x} = 0.$$

Implicitly differentiating with respect to ϕ_0, we get

$$h'(s^*)\frac{\partial s^*}{\partial \phi_0}(\phi_0 + \phi_s s^*) + h(s^*)\left(1 + \phi_s \frac{\partial s^*}{\partial \phi_0}\right) + h(s^*)\phi_s \frac{\partial s^*}{\partial \phi_0} = 0.$$

Therefore,

$$\frac{\partial s^*}{\partial \phi_0} = \frac{-h(s^*)}{h'(s^*)(\phi_0 + \phi_s s^*) + 2h(s^*)\phi_s} > 0,$$

where the inequality follows from Assumption 4. Implicitly differentiating equation (13) with respect to ϕ_s, we get

$$h'(s^*)\frac{\partial s^*}{\partial \phi_s}(\phi_0 + \phi_s s^*) + h(s^*)\left(s^* + \phi_s \frac{\partial s^*}{\partial \phi_s}\right) + h(s^*)\frac{\partial s^*}{\partial \phi_s}\phi_s + H(s^*) = 0.$$

Therefore,

$$\frac{\partial s^*}{\partial \phi_s} = \frac{-h(s^*)s - H(s^*)}{h'(s^*)(\phi_0 + \phi_s s^*) + 2h(s^*)\phi_s} > 0,$$

where again the inequality is a consequence of Assumption 4. Let us next focus on comparative statics with respect to the cost function. Clearly γ_s, γ_s, and $c_x(\cdot)$ do not affect the solution of equation (13). Therefore,

$$\frac{\partial s^*}{\partial \gamma_s} = \frac{\partial s^*}{\partial \gamma_x} = \nabla_{c_x(\cdot)} s^* = 0,$$

(where $\nabla_{c_x(\cdot)}$ denotes the (Gateaux) derivative of the steady-state level of state capacity with respect to the cost function of society).

But the marginal cost of increasing capacity affects the location of the steady state. To quantify this effect, let us implicitly differentiate equation (13) with respect to $c_s'(\delta)$:

$$h'(s^*)\frac{\partial s^*}{\partial c_s'(\delta)}(\phi_0 + \phi_s s^*) + h(s^*)\frac{\partial s^*}{\partial c_s'(\delta)} + h(s^*)\frac{\partial s^*}{\partial c_s'(\delta)}\phi_s = 1.$$

Therefore,

$$\frac{\partial s^*}{\partial c_s'(\delta)} = \frac{1}{h'(s^*)(\phi_0 + \phi_s s^*) + 2h(s^*)\phi_s} < 0.$$

Even though there are unambiguous comparative statics of changes from changes in the output and cost functions on s^* and x^*, it has to be borne in mind that these are the values of state and civil society capacity in a given steady state. The more important conclusion continues to be the

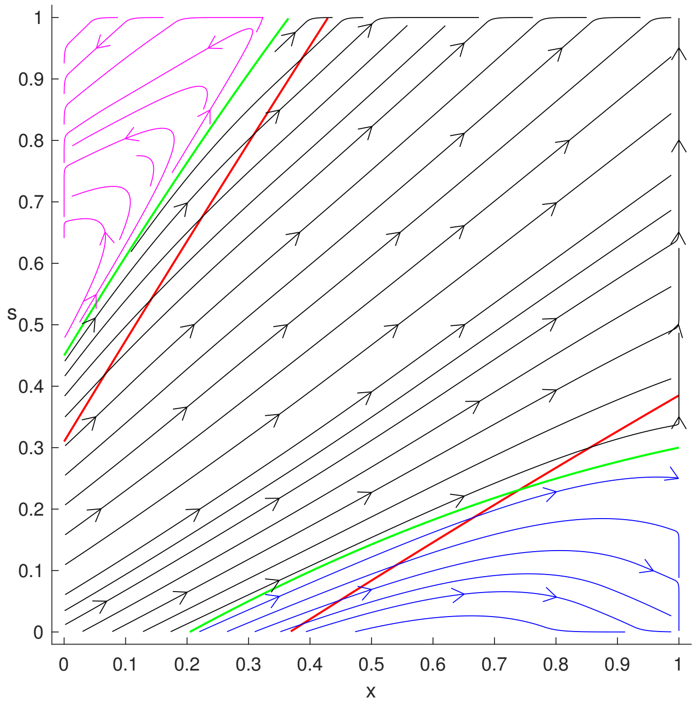

Figure 6: Changes in steady states and dynamics in response to an increase in ϕ_x. The red curves depict the boundaries between the basins of attraction of the different steady states when $\phi_x = 0$ and the green curves show the same boundaries when $\phi_x = 0.1$.

one already highlighted in Proposition 2, that comparative statics in this model are *conditional.* Proposition 2 emphasized this for initial conditions, but they are no less true when we consider changes in the output or cost functions. For instance, an increase in the marginal benefit of the capacity of civil society on output, ϕ_x, increases x^* as we have just shown. However, such a change also shifts the boundaries of the basins of attraction of the different steady states as depicted in Figure 6. As a result, an economy that was previously in Region II — the basin of attraction of the steady state $(1, 1)$ — can now shift to the basin of attraction of the corners steady state $(0, x^*)$ in Region III. Consequently, the long-run state capacity may end up decreasing rather than increasing following an increase in ϕ_x. This reiterates the conclusions of Proposition 2.

5.4 Numerical Results

Figure 6 illustrates how the steady states and the basins of attraction change when we increase ϕ_s, making the capacity of the state more important for overall output. To draw this figure, we use exactly the same parameterization as in the simulation reported in Figure 4, which corresponds to the case in which $\phi_x = \phi_s = 0$ in terms of the model of this section. We then show how the steady states and dynamics are affected when we increase ϕ_x to .1. Particularly noteworthy are the shifts in the boundaries between the regions, which show that the same type of conditional comparative statics highlighted in Section 3.5 in response to shifts in initial conditions now apply when we consider changes in parameters such as the sensitivity of aggregate surplus to the capacity of the state.

In addition, we also show numerically that the same results as those provided above generalize

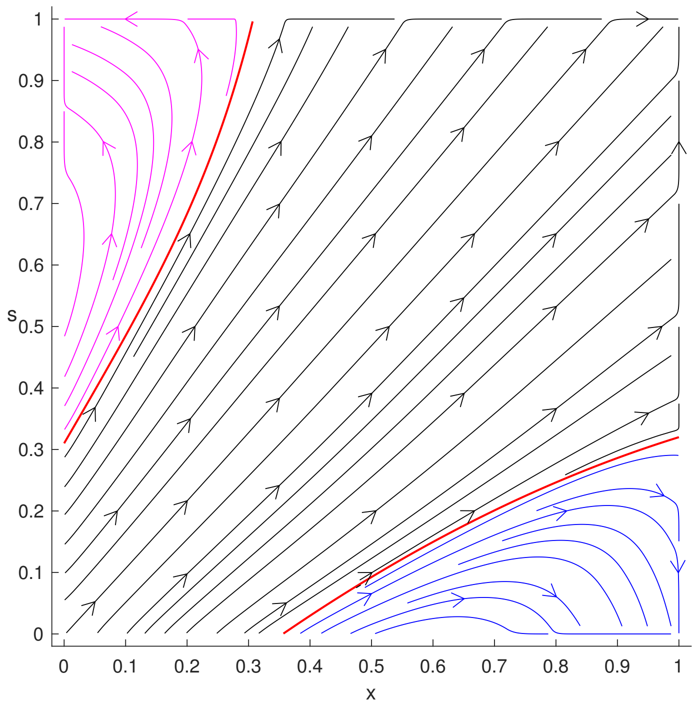

Figure 7: Dynamics when the aggregate output function, $f(x, s)$, is concave.

even when f is concave. Figure 7 depicts the dynamics of state and civil society when we consider the concave surplus function,

$$f(x, s) = .6 + 0.1x^{0.8} + 0.1s^{0.8}.$$

We can see that in this case, the dynamics are very similar to the ones studied in the section where the surplus function is linear.

6 Direct Transitions between Region I and Region III

Figure 2 demonstrates how in our main model, the state space is divided into three regions, and Region II always lies between Regions I and III. However, throughout much of pre-modern history, we have many examples of societies approximating our Regions I and III, but relatively fewer examples of Region II. Perhaps more challengingly for our model, we observe several transitions from Region I directly into Region III, which would not be possible in our baseline model, since Region II is in-between and should be traversed. Here we present a simple modification of the model where Region II shrinks, and creates a subset of the state space (with low levels of state and civil society strength) where Regions I and III are adjacent. The basic idea is to modify the model such that the economies of scale in the cost of investment function becomes dependent on relative strengths.

Suppose that the cost functions for the two players take the form

$$C_x(x_t, x_{t-\Delta}) = c\left(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta\right) + [\max\{\gamma - x_{t-\Delta}, 0\} - \max\{\gamma - s_{t-\Delta}, 0\}]\left(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta\right),$$

and

$$C_s(s_t, s_{t-\Delta}) = c\left(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta\right) + [\max\{\gamma - s_{t-\Delta}, 0\} - \max\{\gamma - x_{t-\Delta}, 0\}]\left(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta\right),$$

where we have made two changes relative to our baseline model. First, we have made c and γ the same for the two players, which is just for simplicity's sake. Second and more important, we have changed the formulation of economies of scale in conflict, so that it is the relative strength of the two players that matters. In particular, when both x and s are less than γ, the second term in the cost function becomes simply a function of the gap between x and s. Clearly this leaves the dynamics when $x_t > \gamma$ and $s_t > \gamma$ unchanged. Consider the case in which $x_t < \gamma$ and $s_t < \gamma$. The differential equations for the strength of society and state can now be written as

$$\begin{aligned} \dot{x} &= (c')^{-1}(h(x-s)+x-s)-\delta \\ \dot{s} &= (c')^{-1}(h(s-x)+s-x)-\delta. \end{aligned}$$

Therefore, defining a new variable $z = x - s$, we have

$$\dot{z} = (c')^{-1}(h(z)+z) - (c')^{-1}(h(z)-z).$$

Or approximating this around $z = 0$, we have

$$\dot{z} = \frac{2z}{c''(\delta)}.$$

Thus regardless of whether $x \gtrless s$, the gap between these two variables will grow, with either x or s increasing. Moreover, with x and s sufficiently small, this implies that we converge to one of these two variables being zero. Therefore, we can conclude that there exists a neighborhood of (0, 0), such that starting in this neighborhood, Region II is absent, and the economy will go to either of the two steady states in Regions I or III. This is depicted in Figure 8, where we use exactly the same parameterization as in Figure 4, except that we use the cost functions in this section and also set $c_x(i) = c_s(i) = 9 \times i^2$, and let $\gamma_x = \gamma_s = 0.4$. This pattern implies that starting with low values of state and civil society strength, a society that starts with a weak state could transition directly into one on the path to a despotic state. However, when we consider societies with sufficiently developed states and civil societies, transitions from despotic or weak states could take us towards an inclusive state.

7 European State Divergence in Historical Perspective

In the Introduction, we discussed the divergent paths of political development in Switzerland, Prussia and Montenegro. In this section, we elaborate on these historical experiences, illustrating how our conceptual framework is useful for interpreting these divergent trajectories. This section has three other related aims. First, we provide evidence about what we mean by the 'strength of society' and how it varied across these societies. Second, we illustrate the notion of conditional comparative statics using the salient example of inter-state warfare. Our model suggests that this has different consequences for the path of state building depending on where a society is in the phase space. This matches closely the historical evidence from these three societies. Finally, we provide evidence that state capacity is greatest not under the despotic path, but when there is contestation between state and society.

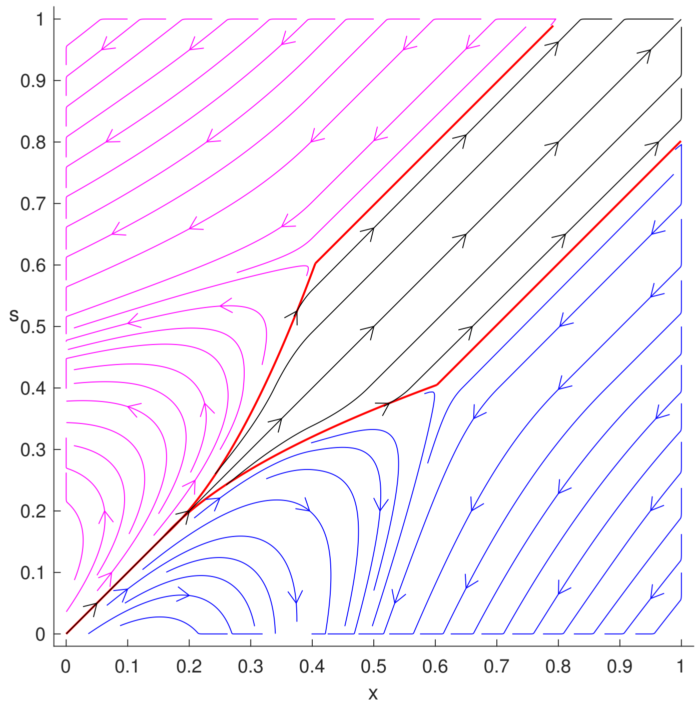

Figure 8: Dynamics with the relative formulation of increasing returns to scale in the cost function for investing in capacity for the state and society. We see that shrinking of Region II and the possibility of direct transitions between Regions I and III.

7.1 Switzerland

Switzerland was historically part of what was the successor political institution to the eastern part of Charlemagne's Carolingian empire; the Holy Roman Empire. The Empire still had an emperor, but it had fragmented into many small and relatively independent polities, and the emperor was actually elected by some of them. Switzerland was on the periphery of the Empire that was centered on Germany. The component polities, the cantons, some mostly rural, some mostly urban, had their own systems of assemblies. These were legacies of two forces, first of the broader pattern of assembly politics inherited from German tribes, which had been institutionalized initially by the Merovingians as the western Roman Empire had fragmented (Wickham, 2017), and second, from the political consequences of the communal economic organization of the medieval period (Blickle, 1998).

The Swiss confederation started in 1291 with men from the cantons of Uri, Schwyz and Unterwalden taking oaths in Rütli, a meadow above Lake Uri and signing the *Bundesbrief* (Federal Charter). The charter was concerned with public order and lawlessness. The first substantive clause read

> "Thus, all people of the valley community of Uri, the entirety of the Schwyz valley and the community of people from the lower Unterwalden valley recognize the malice of the times and for their own protection and preservation they have promised to assist each other by every means possible with every counsel and favor, with persons or goods within their valleys and without against any and all who inflict on them or any among them acts

of violence or injustice against persons or goods."[22]

The Charter was a pact committing the three cantons to come to each other's aid and it provided a framework for resolving disputes. For example, it stipulated that "Should disputes arise among any of the people bound by this oath, the most prudent among the confederates shall settle the conflict between the parties. All other confederates shall defend this verdict against anyone who rejects it." This clause is interpreted as requiring arbitration by one of the cantons if two others, or the citizens of two others, got into a dispute. The Charter therefore was centrally concerned with helping society resolve disputes and cooperate.

We see here two very concrete ways in which the power of society to act collectively was manifested. One was through the direct democracy of the assemblies; "The symbol of Swiss freedom has thus been the *Landesgemeinden* — the cantonal assemblies through which direct participation was assured, and on-going self-government guaranteed" (Barber, 1974, p. 11). The other was the process of oath taking started in 1291. Oaths were taken, and continually re-affirmed, to coordinate and commit adult men to common goals and help solve the collective action problem. This was such a prevalent feature of Switzerland that when the Swiss Confederacy first appeared, it was known to contemporaries as the *Eidgenossenschaft*, the "oath comradeship".

Uri, Schwyz and Unterwalden were in principle subservient to the Habsburg Duke of Austria. The Charter stated "We have further unanimously vowed and established that we in these valleys shall accept no judge who has gained his office for money or for any other price and who is not our resident or native", clearly undercutting the authority of Habsburg judges. The Habsburgs attempted to use force to re-assert control but were defeated in 1315 at the battle of Morgarten. More pacts (and oaths) followed and what came to be known as the Old Swiss Confederation spread. Lucerne joined in 1332. The League of Zurich of 1351 specifically stipulated that everyone who signed would come to the aid of any other threatened by the Habsburgs. Glarus joined in 1352 and Berne in 1353. The Confederacy was continually threatened, for example by Duke Leopold II of Austria, whose army was decisively defeated at the battle of Sempach in 1386 by the combined forces of the Confederation. Leopold was killed along with a lot of the local nobility who had been fighting with him. The Treaty of Basle in 1499 established the Confederation's de facto autonomy.

The Swiss state emerged therefore out of a constant conflict between communes and the collective organization of the cantons, and the higher political institutions of the Holy Roman Empire. After 1291 it was the cantons that accumulated power and took on a collective identity. After 1415 an assembly made up of delegates from all the cantons in the Confederation began to meet regularly. The independent Swiss state was in the making. Fribourg and Solothurn were admitted in 1481, Basel and Schaffhausen in 1501, and Appenzell in 1513.

The independence of the Confederation from the Holy Roman Empire was finally recognized by the Treaty of Westphalia in 1648 which ended the Thirty Years War. The consolidation of a modern central state continued for another 200 years, at least until 1848 after the Sonderbund War where a group of cantons that had opposed strengthening the central state were defeated by a federal army.

[22] https://www.admin.ch/gov/en/start/federal-council/history-of-the-federal-council/federal-charter-of-1291.html

7.2 Prussia

As we noted, the Swiss cantons were part of the Holy Roman Empire. Though Prussia was never part of the Holy Roman Empire, in 1618 it merged through marriage with Brandenburg, which was. The ruling family of Brandenburg, Hohenzollerns, became the ruling family of Brandenburg-Prussia, with the ruler known as the Elector. During the Thirty Years War the newly-united territories were devastated. Brandenburg might have lost as much as half of its population.

In 1640 Frederick William I came to the throne as the new Elector. Known as the Great Elector, Frederick William ruled for 48 years. He charted a new course for Brandenburg-Prussia based on the despotic path of state building. One of his main aims was to build a much more effective military. To achieve this, Frederick William needed tax revenues. Taxes had to be negotiated with various representative bodies, such as the Estates of Kurmark in Brandenburg. He started out by trying to get permanent grants of taxation which would free him from the need to endlessly negotiate them. In 1653 he negotiated the so-called Recess that gave him 530,000 thalers over a period of six years. Crucially, he got to collect the taxes, rather than the Estates. In exchange he gave the nobility, which made up a chamber in the estates, tax-exempt status. This was a clever strategy for dividing the different chambers of the Estates against each other. He went on to extract similar concessions from the Prussian Estates. Frederick William then over-rode the authority of the Estates and started to tax without their agreement something he could start to do because the 1653 decision allowed him to start building a tax bureaucracy. In 1655 he initiated the Kriegskommissariat (the "war commissary") which covered both tax collection and military organization. By 1659 the Estates retreated to local issues, though the institution was never abolished.

Meanwhile Frederick William started to bureaucratize the state. He started with the royal council that was transformed from a set of aristocrats into an administration staffed by professional officials. There was a large pool of well-qualified people with which to fill the new positions. Between 1348 and 1498, for example, 16 universities were founded within central Europe and by 1648 another 18 opened large pool of graduates trained in Roman law who could man a meritocratic bureaucracy. Governors were appointed to run the various territories under the Elector's control. After 1667 he introduced indirect taxes on trade. The administration of the royal estates was also transformed and the land was let out to private farmers for money, dramatically increasing the revenues they generated. By 1688, Brandenburg, Prussia and Kleve-Mark, the largest territories were paying one million thalers a year in taxes in total with about another 600,000 coming in from the other regions under Frederick William's control.

Starting in 1701 Frederick William's son, Frederick III changed Brandenburg-Prussia to the Kingdom of Prussia and crowed himself King Frederick I. His son, confusing called King Frederick William I who ruled between 1713 and 1740 (not to be confused with the Elector of the previous century with the identical name), and his grandson, Frederick II, Frederick the Great, who ruled after 1740, intensified the despotic state-building efforts of the Great Elector.

First in 1723 the bureaucracy was re-organized again with the creation of the General Directory merging the previous war commissary with the administration of the royal estates and putting everything at the service of the military. Then in 1733 Frederick reorganized the basis of military recruitment. He divided the territory into cantons of 5,000 households with a regiment assigned to each for recruitment. At the age of ten every male child was included in the regimental recruitment rolls. Though some occupations and people were exempted at least a quarter of the male population was included in the rolls. The impact of this was to dramatically increase the potential size of the army. In 1713 the army had around 30,000 soldiers in peacetime. By 1740, when Frederick the Great took over from his father, this figure was 80,000. In the meantime his father had managed to increase taxes revenues by almost one half. Frederick the Great had a new strategy for further expanding the tax base and the military machine that Prussia had been building up for a century, he launched an aggressive strategy of territorial expansion. As the French philosopher Voltaire supposedly remarked (he should have known, he was patronized by Frederick the Great)

> "Other states possess an army; Prussia is an army which possesses a state."

The emergence of the modern Prussian state is a classical example of the construction of 'absolutism'. Our main point here is that this very stark difference from Switzerland evolved out of initial circumstances that were far more similar than different. Swiss local autonomy and democracy were not entirely unique; there were similar models all over Germany (Brady, 1985) and the representative institutions of the Estates had considerable powers in many parts of the territories. Yet in Prussia, rather than strenthening, such institutions weakened, allowing the elites move state-building onto a very different path.

7.3 Montenegro

Montenegro in the Balkans had many similarities to Switzerland. It too had been part of the Roman Empire, though more peripherally, and it shared the same type of mountainous ecology and an economy based on herding. Braudel emphasized how European terrain created particular types of societies, arguing that "the mountains are mountains. That is an obstacle. But at the same time a refuge, a land of the free." (Braudel, 1995, p. 39). Indeed, there was a lot of freedom in Montenegro and in similarly-situated Albania too. Edith Durham starts her famous book on the Balkans, *High Albania*, with the line "Of old sat freedom on the heights" (Duham, 1909, p. 1). This freedom was a cause and consequence of the emergence of a weak state in this area.

From the perspective of our model the key to understanding why no state formed in Montenegro was that it was further from the basin of attraction of the inclusive state than Switzerland (or even Prussia) was. It was made up of a group of kinship groups, clans, and lacked the elements of centralization that the Swiss (and Prussians) had inherited from the Carolingians and the Holy Roman Empire. Such kin groups had a great ability to coordinate and they persistently opposed the creation of a state since "Continued attempts to impose centralized government were in conflict with tribal loyalty" (Simić, 1967, p. 87). Society was very strong, while any incipient state was very weak.

As mentioned in the Introduction, prior to 1852 Montenegro was a theocracy, but where the ruling Bishop, the Vladika, could exercise no coercive authority over the clans that dominated the society. After a visit to Montenegro in 1807 the French General Marmont observed "This Vladika is a splendid man, of about fifty-five years of age, with a strong spirit. He conducts himself with nobility and dignity. His positive and legal authority is unrecognized in his country" (Roberts, 2007, p. 174).

The lack of state authority and the dominance of the feud lasted sufficiently long that the anthropologist Christopher Boehm was able to reconstruct it in great detail in the 1960s. Boehm comments on the attempt to exercise central authority that "It was only when their central leader attempted to institutionalize forcible means of controlling feuds that the tribesmen stood firm in their right to follow their ancient traditions. This was because they perceived in such interference a threat to their basic political autonomy" (Boehm, 1986, p. 186). Here Boehm is referring to the attempts of Vladika Njegoš to develop a state in Montenegro in the 1840s. the Montenegrin politician and writer Milovan Djilas describes the situation as "It was a clash between two principles — the state and the clan. The former stood for order and a nation, and against chaos and treason; the latter stood for clan freedoms and against the arbitrary actions of an impersonal central authority - the Senate, the Guard, the captains" (Djilas, 1966, p. 107). Djilas records that Njegoš' reforms were immediately confronted by the revolt of the Piperi and Crmnica clans motivated by the fact that "The imposition of government and a state was putting an end to the independence and internal freedom of the clans" (Djilas, 1966, p. 115). Njegoš was succeeded by his nephew Danilo who made himself the first secular Prince of Montenegro in 1851, but his efforts to centralize authority also ran into fierce opposition. An attempt to raise taxes in 1853 led the clans to revolt with the Piperi, Kuči and Bjelopavlići, declaring themselves an independent state. The attempt failed, and a member of the Bjelopavlići assassinated him in 1860.

Earlier, the first attempt at a codified law in 1796 by Vladika Peter I reflected the fact that order in society was regulated by the institution of blood feuds. It included the clauses: "A man who strikes another with his hand, foot, or chibouk, shall pay him a fine of fifty sequins. If the man struck at once kills his aggressor, he shall not be punished. Nor shall a man be punished for killing a thief caught in the act..." and "If a Montenegrin in self-defense kills a man who has insulted him ... it shall be considered that the killing was involuntary." (quoted in Durham, 1928, pp. 78-88). Djilas describes the importance of blood feuds in the 1950s thus "the men of several generations have died at the hands of Montenegrins, men of the same faith and name. My father's grandfather, my own two grandfathers, my father, and my uncle were killed, as though a dread curse lay upon them ... generation after generation, and the bloody chain was not broken. The inherited fear and hatred of feuding clans was mightier than fear and hatred of the enemy, the Turks. It seems to me that I was born with blood in my eyes. My first sight was of blood. My first words were blood and bathed in blood" (1958, pp. 3-4).

Though Montenegro had many similarities with Switzerland, economically, ecologically and even in terms of the underlying clan structure of society, it lacked any legacy of political centralization and thus was in the basin of attraction of the weak state.

7.4 Conditional Comparative Statics

One of the most famous theories linking the emergence of state capacity to structural factors is about the role of warfare. As Tilly famously put it, "War made the state, and the state made war". Indeed, warfare was ever present in all of these three cases, but with very different consequences.

For the Swiss, the threat of warfare, in particular the persistent threat from the Habsburgs to reinstate the over-lordship of the Holy Roman Empire, seems to have been an important incentive for the otherwise individualistic cantons and cities to unite into a larger confederation and to centralize authority and decision-making. This seems to have overcome what would otherwise have been strong centrifugal tendencies. As one historian put it the Swiss peasants were "free of feudal servitudes and, as a sign of their liberty, these mountain peasants bore arms and demanded 'honor' even from nobles ... Their medieval clan structures had little to do with our images of democratic forms but these peasants were 'free"' (Steinberg, 2015, p. 18). Swiss mercenaries ended up fighting in most European armies, and as early as 1424 the Assembly of the Confederation was asked by Florence to supply them with mercenaries.

Thus the centralization that started in 1291 was likely encouraged by the pressure of warfare. As Machiavelli put it in *The Prince*, published in 1513, "Rome and Sparta for many centuries stood armed and free. The Swiss are extremely well armed and are very free". In terms of our phase diagram we can therefore think of warfare pushing Switzerland, a very decentralized polity of quasi-democratic cantons, in a more centralized direction and into the basin of attraction of the inclusive state.

The outcome in Prussia was very different. Though Brandenberg had many of the same structural features of Switzerland and Prussia was run by its ruling house, the territories to the east were a world apart. They did not have the same history of communes and quasi-democratic politics like the Swiss cantons. Civil society was weaker and more easily dominated. Nevertheless, at the start of the early modern period it is plausible to believe that Prussia was in, or at the very least in the vicinity of, the basin of attraction of the inclusive state. In this light, the impact of Thirty Year War can be thought of as forcing Prussia out of the basin of attraction of the inclusive state and into that of the despotic path. This is how Frederick the Great himself viewed the situation:

> "So long as God gives me breath, I shall assert my rule like a despot ... A well-run government must have a firmly established system ... in which finance, policy and military all combine to promote the same end, the strengthening of the state and the expansion of its power. Such a system can only derive from one brain" (quoted in Blanning, 2015, p. 127)

The British envoy Hugh Elliot's assessment was "The Prussian monarchy reminds me of a vast prison in the center of which appears the great keeper occupied in the care of his captives" (quoted in Blanning, 2015, p. 146). Warfare pushed Prussia off a path to an inclusive state. A very different outcome to the one warfare helped create in Switzerland.

Finally, war did not make the state in Montenegro (nor in neighboring Albania) and the incentives it created were not powerful enough to move the country out of the basin of attraction of the weak state. Like Switzerland, Montenegro had clans, but these had never been integrated into the type of centralized institutions that the Holy Roman Empire had created. Thus power was very decentralized. The impact of continuous warfare with the Ottomans did induce the clans to try to coordinate more and create more central structures. Just before a key battle at Krusi in 1796, an assembly of Monetnegrin tribal chiefs met at Cetinje and adopted a measure called Stega, or 'fastening' which proclaimed the unification of the Montenegrin heartlands. Two years later they met and agreed to the creation of a 'council' of 50 members, in effect the first time there had been any institutionalized structures of government above the tribe. As we have seen however, this impulse was not sufficiently powerful to create a state.

7.5 The Capacity of the State

One of the most surprising implications of our analysis is that it is not the despotic state which has more capacity, but the inclusive one. This is evident from the English case we discussed in the Introduction. The bottom-up process of state development, full of competition and contestation between state and society, led to the development of a very high-capacity state in England and then Britain as emphasized by, among others, Braddick (2000), Hindle (2002) and Tilly (1995). In his comparative study of the development of European states, Ertman (1997) emphasizes that it was the English state that developed the greatest capacity.

The same conclusion follows from the comparison of Switzerland, Prussia and Montenegro as well. This can be seen when we focus on the early modern period, before the defeat of Prussia by Napoleonic armies at Jena in 1806 triggered radical institutional reform. This is a period before mass education, massive investments in infrastructure, or social insurance. We therefore focus on perhaps the most basic public good that a state should provide, dispute resolution and law enforcement.

Prussia is typically referred to as an example of 'bureaucratic absolutism' and we discussed above the emergence of a fiscal-military bureaucracy after 1648. Nevertheless, this bureaucracy never penetrated into the lives of most Prussians. In fact, it floated ontop of a large feudal infrastructure which was very far from being bureaucratic. Rosenberg showed how there "developed an untidy and disjointed amalgamation of the new merit system with the old "spoils system"'. And "the latter was sustained by aristocratic patronage, social heredity, amateurism, and, often, proprietary tenure" (1958, p. 75).[23] Local institutions, including the manorial courts, were completely controlled by aristocrats and elites who used them to repress and coerce serfs in a society that was largely rural, with as much as 80% of the population working in agriculture.[24] was run by local elites and aristocrats who used it, particular legal institutions, to repress and control serfs.

> "The noble estate became an integrated legal and political space. The landlord was not only the employer of his peasants and the owner of their land; he also held jurisdiction over them through the manorial court, which was empowered to issue punishments ranging from small fines for minor misdemeanors to corporal punishments, including whippings and imprisonment." (Clark, 2009, pp. 161-162).

A contemporary commentator noted in 1802

[23]These systems were indeed infused with heavy patrimonial elements. For example, "several consecutive generations of the upper levels of the ... Prussian mining administration ... were completely dominated by a tiny, intermarried group of firmly entrenched noble and nonnoble families. Wherever in the Prussian economy salt and coal mining, iron making, and the metallurgical industry [were important one found that] the Heinitzs, von Redens, von Hardenbergs, von Steins, Dechens, Gerhards, and their kinsmen were prominent" (Rosenberg, 1958, p. 82). Ziblatt (2009) shows that this patrimonialism persisted right up until the end of the 19th century.

[24]See Clark (2009, p. 160). In Brandenburg, in fact, around 60% of the land was owned in feudal tenure (Cerman, 2012). Though a revisionist literature (e.g., Hagan, 2002, Eddie, 2012) claims that feudal lordship was not as onerous as traditional accounts maintained, there is broad agreement on the extent of legal control by the lords, their extensive role in any type of dispute resolution (of course not impartially), and the widespread existence of unpaid labor services, restrictions on mobility and other controls and fines that they were able to impose on peasants via the manorial courts they controlled (e.g., Carstens, 1954, Rosenberg, 1958, Cerman, 2012).

"laws cannot relieve the suffering of the serf. For where should he sue? To the manorial court which, by its very nature, stood under the influence of the lord? Or to the provincial government, which ... remained inaccessible to him?" (quoted in Eddie, 2013, p. 31)

A major consequence of this structure was that the capacity of the state was highly limited at the local level and there was a dearth of public good provision and central control. In his study of the administration of early modern ordinances in Germany, Raeff noted "it was no so much a sovereign's or political entity's ability to centralize and monopolize power that mattered for the successful implementation of its ordinances. It was rather a government's ability to reach out to local communities and subordinate institutions, through effective channels of communication, that proved crucial. In those instances when the sovereign succeeded in enlisting the cooperation of lower, preexisting associations, solidarities or institutions, the police ordinances had the best chance of being effective" (1983, p. 45-46). In contrast to the English case discussed in the Introduction, the ability of the Prussian elite to achieve cooperation with the weakened society was limited, and this curtailed the capacity of the Prussian state. Further evidence of the frail capacity of the Prussian state, as well as other German states during this period, is emphasized by Raeff, when he states that particularly in "Brandendurg and Magdeberg in Prussia" the "same regulations and prescriptions are repeated constantly and also echoed over long periods of time within the same political jurisdiction" because "the original ordinance was not properly obeyed or implemented" (Raeff, 1983, p. 51).

The contrast between this Prussian picture and the Swiss case is stark. As we showed above, the Swiss confederacy was founded on a demand for the objective resolution of legal disputes (which feudal Habsburg courts rarely delivered). At the commune level, magistrates were elected and Switzerland was constructed by a series of covenants and pacts which recognized the autonomy and self-governance of local society. Swiss society then fought a long, and successful battle, against precisely the type of local despotism that Prussian peasants had to put up with.

In Montenegro, though the evidence for the early modern period is thin, we know that there were no real legal institutions and disputes were mediated by the clans and the feud. This represents much less capacity than the Swiss case who were able to create an institutionalized system of law and justice.

8 Conclusion

There is a great deal of diversity in the nature of states in the world today, in particular in the extent to which they have capacity to fulfill basic functions, such as raise tax revenues, establish a monopoly of violence or effectively regulate society. But societies, not just states, also differ enormously. Some are highly mobilized and organized collectively, with high levels of 'social capital', strong norms against political hierarchy, and later institutions that facilitate collective actions. In contrast, in others non-elites are weak and incapable of contesting for power against elites and the state. The capabilities of states and societies go together. When the state and elites are strong, societies tend to be subservient and weak, and when societies are well organized and capable of resisting political hierarchy, the states tend to remain weak. Consequently, and strikingly, historical evidence and contemporary cases also suggest that states with the greatest capacity often emerge when there is an ongoing competition between states and societies — so that we see the greatest power of the state not when it can despotically dominate society, but when it is matched by the society's power.

In this paper we have developed a simple model to understand the variation in state capacity, arguing that states endogenously acquire capacity in a dynamic contest with society. At the heart of our model is the notion that elites that control states must contest with society (non-elites) for control over political power, resources and rents. If the state accumulates capacity, then this helps it win this contest. But in response society can also accumulate strength, and this contestation from society in turn encourages the state to build further capacity. In our model, this logic leads to three distinct stable steady states with very different constellations of state society relations. In one steady state, which we called despotic, the state acquired far more strength than society, in a sense dominating it. In the reverse situation, where society accumulates more strength than the state, we have a weak state. Finally, and arguably most interestingly, a rough balance of power between state and society leads to the emergence of an inclusive state. Our model clarifies how the competition between state and society in this case is the engine behind the emergence of the greatest state capacity. Despotic states, because they can easily dominate society, have less reason to accumulate as much power and capacity.

We view our paper as a first step in the investigation of the competition and cooperation between state and various different segments of society. Several directions of research are exciting in this context. First, providing further microfoundations for the competition between elites (state) and non-elites (society) is an obvious area of research. Second, one could also model the cooperation between state and society more systematically, taking into account the dynamic interactions and the emergence of trust and legitimacy of state institutions (see Acemoglu, 2005, for a preliminary attempt in this direction). Third, it would be interesting to allow for the rich heterogeneity that exists between different groups in society as well as the competition within the elite. Fourth, a major topic for research is to investigate how state institutions start gaining their independence and autonomy from elite interests. Fifth, there is also much more to be done in the investigation of the role of various different structural factors, including how they may shift the economy into the basins of attraction of different types of steady states. Last but not least, the perspective presented here opens the way for new types of empirical research, studying the extent of history dependence and potentially discontinuous divergence from similar initial conditions in state-society relations.

Appendix

Proof of Lemma 1. At $x^* = s^* = 1$, the marginal cost of investment for player $z \in \{x, s\}$ is $c'_z(\delta)$, while the marginal benefit starting from this point is $h(0)$, so Assumption 3 ensures that the marginal benefit strictly exceeds the marginal cost, and neither player has an incentive to reduce its investment. Furthermore, because 1 is the maximum level of investment, neither party has the ability to increase it.

We turn next to asymptotic stability of this steady state. First note that from (7), the laws of motion of x and s in the neighborhood of the state state ($x^* = 1, s^* = 1$) are given by

$$\begin{aligned} c_x'(\dot{x} + \delta) &= h(x - s) \text{ if } x < 1 \text{ and } \dot{x} = 0 \text{ if } x = 1 \\ c_s'(\dot{s} + \delta) &= h(s - x) \text{ if } s < 1 \text{ and } \dot{s} = 0 \text{ if } s = 1, \end{aligned} \tag{14}$$

where we are exploiting the fact that once we are away from the steady state, there cannot be an immediate jump and thus the first-order conditions have to hold in view of Assumption 2. We have also used the information that we are in the neighborhood of the steady state $(1, 1)$ in writing the system for $x > \gamma_x$ and $s > \gamma_s$. Now to establish asymptotic stability, we will show that

$$L(x, s) = \frac{1}{2}(1 - x)^2 + \frac{1}{2}(1 - s)^2$$

is a Lyapunov function in the neighborhood of the steady state $(1, 1)$. Indeed, $L(x, s)$ is continuous and differentiable, and has a unique minimum at $(1, 1)$. We next verify that in a sufficiently small neighborhood of $(1, 1)$, $L(x, s)$ is decreasing along solution trajectories of the dynamical system given by (14). Since L is differentiable, for $x \in (\gamma_x, 1)$ and $s \in (\gamma_s, 1)$, we can write

$$\frac{dL(x, s)}{dt} = -(1 - x)\dot{x} - (1 - s)\dot{s}.$$

First note that since $h(x - s) > c_x'(\delta)$ and $h(s - x) > c_s'(\delta)$ for x and s in a sufficiently small neighborhood of $(1, 1)$, we have both $\dot{x} > 0$ and $\dot{s} > 0$. This implies that, in this range, both terms in $\frac{dL(x,s)}{dt}$ are negative, and thus $\frac{dL(x,s)}{dt} < 0$. Moreover, the same conclusion applies when $x = 1$ (respectively when $s = 1$), with the only modification that $\frac{dL(x,s)}{dt}$ no longer includes the $\dot{s}$ (respectively the $\dot{x}$) term, but still continues to be strictly negative, even on the boundary of $[0, 1]^2$. Then the asymptotic stability of $(1, 1)$ follows from LaSalle's Theorem (which takes care of the fact that our steady state is on the boundary of the domain of the dynamical system in question, see, e.g., Walter, 1998). ■

Proof of Lemma 2. We start with the first statement. Suppose first that $x^* = 0$. Then from (6) an interior steady-state level of investment for the state requires

$$h(s) = c_s'(\delta) + \max\{0; \gamma_s - s\}.$$

Note that Assumption 3 implies that at $s = 1$, $h(1) < c_s'(\delta)$, and at $s = \gamma_s$, $h(\gamma_s) > c_s'(\delta)$, thus by the intermediate value theorem, there exists s^* between γ_s and 1 satisfying

$$h(s^*) = c_s'(\delta). \tag{15}$$

Moreover, because h is single peaked and symmetric around 0, $h(s)$ is decreasing in $s \geq \gamma_s$, and thus only a unique s^* satisfying this relationship exists.

We next verify that $x^* = 0$ is indeed consistent with the optimization of civil society. This follows immediately since

$$h(-s^*) = h(s^*) = c_s'(\delta) < c_x'(0) + \gamma_x,$$

where the first equality follows from the symmetry of h, the second one simply replicates (15), and the inequality follows from Assumption 2, and establishes that $x^* = 0$ is optimal for civil society.

The local stability is again established using a Lyapunov argument as in the proof of Lemma 1. Now in the neighborhood of the state state $(x = 0, s = s^*)$, the dynamical system in (7) can be written as

$$
\begin{aligned}
c_x'(\dot{x} + \delta) &= h(x - s) + \gamma_x - x \text{ if } x > 0 \text{ and } \dot{x} = 0 \text{ if } x = 0, \text{ and} \\
c_s'(\dot{s} + \delta) &= h(s - x),
\end{aligned}
$$

where we are now using the fact that we are in the neighborhood of $(0, s^*)$ so that $x < \gamma_x$ and $s > \gamma_s$. The dynamical system in (7) in this case can be written as

$$
\begin{aligned}
\dot{x} &= (c_x')^{-1}(h(x - s) + \gamma_x - x) - \delta \\
\dot{s} &= (c_s')^{-1}(h(s - x)) - \delta.
\end{aligned} \tag{16}
$$

We now choose the Lyapunov function

$$L(x, s) = \frac{1}{2}x^2 + \frac{1}{2}(s - s^*)^2,$$

which is again continuous and differentiable, and has a unique minimum at $(0, s^*)$. We next verify that in the neighborhood of $(0, s^*)$, $L(x, s)$ is decreasing along solution trajectories of the dynamical system given by (16). Specifically, since L is differentiable, for $x \in (0, \gamma_x)$ and $s \in (\gamma_s, 1)$, we can write

$$\frac{dL(x, s)}{dt} = x\dot{x} + (s - s^*)\dot{s}.$$

First note that as $h(-s^*) < c_x'(\delta) + \gamma_x$, for x and s in the neighborhood of $(0, s^*)$,

$$\dot{x} = (c_x')^{-1}(h(x - s) + \gamma_x - x) - \delta < 0. \tag{17}$$

Then, using a first-order Taylor expansion of (16) in this neighborhood, we obtain

$$(s - s^*)\dot{s} = \frac{1}{c_s''(\delta)}h'(s^*)(s - s^*)(s - x - s^*) + o(\cdot), \tag{18}$$

where $o(\cdot)$ denotes second-order terms in x and $s - s^*$.

The desired result follows from the following arguments: (i) for $x \in (0, \gamma_x)$ and $s \in (\gamma_s, 1)$, $|x\dot{x}| > |(s - s^*)\dot{s}|$, regardless of the sign of $(s - s^*)\dot{s}$, as $x \to 0$ and $s \to 0$, $(s - s^*)(s - x - s^*)/x \to 0$, because in the neighborhood of the steady state $(0, s^*)$, $\dot{s}$ is of the order $s - s^*$, while $h(-s^*) < c_x'(\delta) + \gamma_x$, ensuring that $\dot{x} < 0$). Therefore, in the range where $x \in (0, \gamma_x)$ and $s \in (0, \gamma_s)$, $\frac{dL(x,s)}{dt} < 0$. (ii) when $x = 0$, (18) implies that $(s - s^*)\dot{s} < 0$ in view of the fact that $h'(s^*) < 0$, and thus we have $\frac{dL(x,s)}{dt} < 0$. (iii) when $s = s^*$, (17) ensures that $\dot{x} < 0$, so that we again have $\frac{dL(x,s)}{dt} < 0$. Then in all three cases, the asymptotic stability of $(0, s^*)$ follows from LaSalle's Theorem (e.g., Walter, 1998).

The proof of the existence, uniqueness and asymptotic stability of the steady state with $s^* = 0$ and $x^* \in (\gamma_x, 1)$ is analogous, and is omitted. ■

Proof of Lemma 3. Claim (i) follows immediately, since from part 3 of Assumption 3, we have $h(0) - \gamma_s > c_s'(0)$, so that when $x^* = 0$, the elite will deviate from $s = 0$. Claim (ii) follows directly from the proof of Lemma 2. Finally, for claim (iii), note that a steady state with $x^* \in (\gamma_x, 1)$ and $s^* \in (\gamma_s, 1)$ would necessitate

$$\begin{aligned} h(s^* - x^*) &= c_s'(\delta) \\ h(x^* - s^*) &= c_x'(\delta), \end{aligned} \tag{19}$$

but then from the symmetry of the h function around zero, we have that $h(s^* - x^*) = h(x^* - s^*)$, so that

$$c_s'(\delta) = h(s^* - x^*) = c_x'(\delta),$$

which contradicts part 2 of Assumption 2. ■

Proof of Lemma 4. We will prove this lemma by considering three types of steady states, which exhaust all possibilities.

Type 1: $x^* \in (0, \gamma_x)$ and $s^* \in (0, \gamma_s)$.

The optimality conditions in such a steady state are

$$\begin{aligned} h(s^* - x^*) &= c_s'(\delta) + \gamma_s - s^* \\ h(x^* - s^*) &= c_x'(\delta) + \gamma_x - x^*. \end{aligned}$$

The dynamical system (7) now becomes

$$\begin{aligned} \dot{x} &= (c_x')^{-1}(h(x^* - s^*) + \gamma_x - x^*) - \delta \\ \dot{s} &= (c_s')^{-1}(h(s^* - x^*) + \gamma_s - s^*) - \delta. \end{aligned}$$

Since the steady-state levels of state and civil society strength are defined by equality conditions in this case, local dynamics can be determined from the linearized system, with characteristic matrix given by

$$\begin{pmatrix} \frac{1}{c_s''(\delta)}[h'(s^* - x^*) + 1] & -\frac{1}{c_s''(\delta)} h'(s^* - x^*) \\ -\frac{1}{c_x''(\delta)}[h'(x^* - s^*) & \frac{1}{c_x''(\delta)}[h'(x^* - s^*) + 1] \end{pmatrix}.$$

Using the fact that from Assumption 3, $h'(s^* - x^*) = -h'(x^* - s^*)$, the determinant of this matrix can be computed as $\frac{1}{c_s''(\delta) c_x''(\delta)} > 0$. Moreover, from part 2 of Assumption 2, we can show that the trace of this matrix is

$$\frac{1}{c_s''(\delta)}[h'(s^* - x^*) + 1] + \frac{1}{c_x''(\delta)}[h'(x^* - s^*) + 1].$$

Once again using Assumption 3, this expression is positive provided that

$$h'(s^* - x^*)(c_s''(\delta) - c_x''(\delta)) \leq c_x''(\delta) + c_s''(\delta). \tag{20}$$

Assumption 2 ensures that

$$\left| c_s''(\delta) - c_x''(\delta) \right| \leq \frac{c_x''(\delta)}{|h'(s^* - x^*)|},$$

which is a sufficient condition for (20), establishing that both eigenvalues are positive, and we have asymptotic instability.

Type 2: $x^* \in (\gamma_x, 1)$ and $s^* \in (0, \gamma_s)$, or $x^* \in (0, \gamma_x)$ and $s^* \in (\gamma_s, 1)$. Consider the first of these,

$$\begin{aligned} h(s^* - x^*) &= c_s'(\delta) + \gamma_s - s^* \\ h(x^* - s^*) &= c_x'(\delta). \end{aligned}$$

Now once again, local dynamics can be determined from the linearized system, with characteristic matrix

$$\begin{pmatrix} \frac{1}{c_s''(\delta)}[h'(s^*-x^*)+1] & -\frac{1}{c_s''(\delta)}h'(s^*-x^*) \\ -\frac{1}{c_x''(\delta)}[h'(x^*-s^*) & \frac{1}{c_x''(\delta)}h'(x^*-s^*) \end{pmatrix}.$$

The trace of this matrix is

$$\frac{1}{c_s''(\delta)}[h'(s^*-x^*)+1]+\frac{1}{c_x''(\delta)}h'(x^*-s^*),$$

which is positive provided that

$$h'(s^*-x^*)(c_s''(\delta)-c_x''(\delta)) \leq c_x''(\delta).$$

The same argument as in the proof of Type 1 shows that this condition follows from Assumption 2, implying that at least one of the eigenvalues is positive and thus establishing asymptotic instability. The argument for the case where $x^* \in (0,\gamma_x)$ and $s^* \in (\gamma_s, 1)$ is analogous.

Type 3: $s^* = 1$ and $x^* < 1$ or $x^* = 1$ and $s^* < 1$.

We prove the first case (the proof for the second is analogous). Such a steady state exists only if

$$\begin{aligned} h(1-x^*) &\geq c_s'(\delta) \\ h(x^*-1) &= c_x'(\delta)+\max\{\gamma_x - x^*, 0\}. \end{aligned}$$

Exploiting these conditions, we will show that such a steady state cannot be asymptotically stable. To do this, let us distinguish between $x^* > \gamma_x$ and $x^* \leq \gamma_x$. Consider the first one of these. Then consider a perturbation that keeps s^* constant and reduces x^* to $x^* - \varepsilon_x$ for $\varepsilon_x > 0$ small (since it is sufficient to show asymptotic instability for a specific set of perturbations). Then, we have

$$\dot{x} = -\frac{1}{c_x''(\delta)}h'(x^*-1)-\delta < 0.$$

The sign follows because $h'(x^*-1) > 0$ from Assumption 3, and implies that x^* decreases away from the steady state in question, establishing asymptotic instability. Consider finally the second possibility, with the same perturbation which yields

$$\dot{x} = -\frac{1}{c_x''(\delta)}[h'(x^*-1)+1]-\delta < 0,$$

which is also locally asymptotically unstable. This completes the proof of the lemma. ■

Proof of Lemma 6. This follows given the continuous differentiability of $V_x(x,s,\beta;\Delta)$ and $V_s(x,s,\beta;\Delta)$ and of $x'^*(x,s,\beta;\Delta)$ and $s'^*(x,s,\beta;\Delta)$ for all $\Delta > 0$. ■

Proof of Proposition 4. The proof of this proposition follows directly from the proofs of Propositions 1 and 3, with only minor changes to Lemma 4, which we provide next, ruling out the stability of three different types of steady states. We again treat each type separately.

Type 1: $x \in (0,\gamma_x)$ and $s \in (0,\gamma_s)$.

The optimality conditions in such a steady state are

$$\begin{aligned} h(s-x)(\phi_0+\phi_x x+\phi_s s)+H(s-x)\phi_s &= c_s'(\delta)+\gamma_s - s \\ h(x-s)(\phi_0+\phi_x x+\phi_s s)+H(x-s)\phi_x &= c_x'(\delta)+\gamma_x - x. \end{aligned}$$

Local dynamics are in turn given by

$$\begin{aligned} h(s-x)(\phi_0+\phi_x x+\phi_s s)+H(s-x)\phi_s &= c_s'(\dot{s}+\delta)+\gamma_s-s \\ h(x-s)(\phi_0+\phi_x x+\phi_s s)+H(x-s)\phi_x &= c_x'(\dot{x}+\delta)+\gamma_x-x. \end{aligned}$$

Since the steady-state levels of state and civil society strength are defined by equality conditions in this case, local dynamics can be determined from the linearized system, with characteristic matrix given by

$$\begin{pmatrix} \frac{1}{c_s''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_s+1] & \frac{1}{c_s''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+h(\cdot)(\phi_x-\phi_s)] \\ \frac{1}{c_x''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+h(\cdot)(\phi_s-\phi_x)] & \frac{1}{c_x''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_x+1] \end{pmatrix},$$

where we wrote $h(\cdot)$ or $h'(\cdot)$ instead of $h(s-x)$ and $h'(s-x)$ in order to save space (and we will adopt this shorthand whenever we write matrices or long expressions below). From part 2 of Assumption $3'$, we can show that the trace of this matrix is positive. In particular, the trace is given by

$$\frac{1}{c_s''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_s+1]+\frac{1}{c_x''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_x+1].$$

Using Assumption $3'$, this expression is positive if

$$h'(s-x)(c_s''(\delta)-c_x''(\delta))(\phi_0+\phi_x x+\phi_s s)\leq(c_x''(\delta)+c_s''(\delta))(1+2h(s-x)(\phi_s+\phi_x)). \tag{21}$$

Assumption $2'$ ensures that

$$\left|c_s''(\delta)-c_x''(\delta)\right|\leq\frac{c_x''(\delta)(1+2h(s-x)(\phi_s+\phi_x))}{|h'(s-x)|\,(\phi_0+\phi_x+\phi_s)},$$

which is a sufficient condition for (21), establishing that at least one of the eigenvalues is positive, and we have asymptotic instability.

Type 2: $x\in(\gamma_x,1)$ and $s\in(0,\gamma_s)$, or $x\in(0,\gamma_x)$ and $s\in(\gamma_s,1)$. Consider the first of these,

$$\begin{aligned} h(s-x)(\phi_0+\phi_x x+\phi_s s)+H(s-x)\phi_s &= c_s'(\delta)+\gamma_s-s \\ h(x-s)(\phi_0+\phi_x x+\phi_s s)+H(x-s)\phi_x &= c_x'(\delta). \end{aligned}$$

Now, once again, local dynamics can be determined from the linearized system, with characteristic matrix

$$\begin{pmatrix} \frac{1}{c_s''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_s+1] & \frac{1}{c_s''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+h(\cdot)(\phi_x-\phi_s)] \\ \frac{1}{c_x''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+h(\cdot)(\phi_s-\phi_x)] & \frac{1}{c_x''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_x] \end{pmatrix}.$$

The trace of this matrix can now be computed as

$$\begin{aligned} &\frac{1}{c_s''(\delta)}[h'(s-x)(\phi_0+\phi_x x+\phi_s s)+2h\phi_s+1] \\ &+\frac{1}{c_x''(\delta)}[h'(x-s)(\phi_0+\phi_x x+\phi_s s)+2h(x-s)\phi_x]. \end{aligned}$$

which is positive if

$$h'(s-x)(c_s''(\delta)-c_x''(\delta))(\phi_0+\phi_x x+\phi_s s)\leq(c_x''(\delta)+c_s''(\delta))(2h(s-x)(\phi_x+\phi_s))+c_x''(\delta).$$

The same argument as in the proof of Type 1 establishes that this condition follows from Assumption 2', and thus at least one of the eigenvalues is positive and the steady state in question is asymptotically unstable. The argument for the case where $x \in (0, \gamma_x)$ and $s \in (\gamma_s, 1)$ is analogous.

Type 3: $x = 1$ and $s < 1$ or $s = 1$ and $x < 1$.

Let us prove the first case. Such a steady state would require

$$\begin{aligned} h(1-s)(\phi_0 + \phi_x + \phi_s s) + H(1-s)\phi_x &\geq c_x'(\delta) \\ h(s-1)(\phi_0 + \phi_x + \phi_s s) + H(s-1)\phi_s &= c_s'(\delta) + \max\{0, \gamma_s - s\}. \end{aligned}$$

We distinguish between $s \leq \gamma_s$ and $s > \gamma_s$. Consider the first one of these. Consider a perturbation to $s + \varepsilon_s$ for $\varepsilon_s > 0$ (it is sufficient to consider perturbations that maintain x constant). Then the local dynamics of s are given by:

$$\dot{s} = \frac{1}{c_s''(\delta)}[h'(s-1)(\phi_0 + \phi_x + \phi_s s) + 2h(s-1)\phi_s + 1]\varepsilon_s.$$

From Assumption 3$'$, $h'(s-1) > 0$, the conflict capacity of the state locally diverges from this steady state, establishing asymptotic instability. Consider next the second possibility. In this case, for $s + \varepsilon_s$, we have

$$\dot{s} = \frac{1}{c_s''(\delta)}[h'(s-1)(\phi_0 + \phi_x + \phi_s s) + 2h(s-1)\phi_s]\varepsilon_s,$$

which is also locally asymptotically unstable. The other case is proved identically. ■

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