Derivation of the Stiglitz - Shapiro Model

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1 Variable Definitions

- Monitoring workers effort is costly
- Real wage as incentive to work
- Workers Effort affected by cost and probability of Unemployment
- The Market Equilibrium shows involuntary unemployement

N = Number of Workers, L Number of workers Employed, $u = \frac{N-L}{l}$ = unemployment rate
w = wages, $\overline{w}$ = unemployment benefits
b = probability to quit the job, a = job acquisition rate
q = probability of being detected shirking
V_E^N = *Expected* Utility of Being Employed and Non Shirking
V_E^S = *Expected* Utility of Being Employed and Shirking
V_u = *Expected* Utility of Being Unemployed

1.0.1 Workers

Workers choose the level of effort e that maximises the intertemporal utility function

$$U = \int_0^{\infty} u\left(w\left(t\right), e\left(t\right)\right) \exp^{-rt} dt$$

Worker faces a constant probability of unemployment b, which provide an unemployment benefit $\overline{w}$. If worker shirks, the probability of being caught and fired is q

Utility of a Non-Shirker Worker

$$rV_E^N = w - e + b\left(V_u - V_E^N\right) \rightarrow V_E^N = \frac{(w-e) + bV_u}{r+b} \tag{1}$$

Utility of a Shirker Worker

$$rV_E^s = w + (b+q)\left(V_u - V_E^s\right) \rightarrow V_E^s = \frac{w + (b+q)\,V_u}{r+b+q} \tag{2}$$

1.0.2 NO SHIRKING CONDITIONS

$$V_E^N \geq V_E^s$$

Sufficient Condition - the risk taken in shirking should be higher than the extra effort required by non-shirking

$$q\left(V_E^s - V_u\right) \geq e$$

Wage rate required for not shirking:

$$w \geq rV_u + (r+b+q)\, e/q \equiv \widetilde{w}$$

- Critical wage funtion of the and probability of unemployment
- More costly monitoring implies higher non-shirking wage

1.0.3 Employers

M identical firms ($i = 1..M$), with $q_i = f_i\left(L_i\right)$

The Firm has to decide the wage and the quantity of labour that maximise productivity of labour, given cost of monotiring q assumed exogenous

Aggregate Production Function	$Q = F(L)$
Aggregate Labour Demand	$F' = \widetilde{w}$

Market Equilibrium

High Wages, High Unemployment, High Effort
Low Wages, Low Unemployment, Low Effort

Equlibrium Expected Utility for an Employed Worker ($=V_E^N$)

$$rV_E = w - e + b\left(V_u - V_E\right)$$

Expected Utility of an Unemployed Worker

$$rV_u = \overline{w} + a\left(V_E - V_U\right)$$

Solving Simultaneously

$$V_u = \frac{\overline{w} + aV_E}{(r+a)}$$

$$\begin{aligned}
rV_E &= w - e + b\left(\frac{\overline{w} + aV_E}{(r+a)} - V_E\right) = w - e + b\left(\frac{\overline{w} - rV_E}{(r+a)}\right) \\
rV_E + \frac{brV_E}{(r+a)} &= w - e + b\left(\frac{\overline{w}}{r+a}\right) \\
\frac{r+a+b}{(r+a)}rV_E &= w - e + b\left(\frac{\overline{w}}{r+a}\right) \\
rV_E &= \frac{r+a}{r+a+b}\left(w - e + b\left(\frac{\overline{w}}{(r+a)}\right)\right) = \frac{(r+a)(w-e) + b\overline{w}}{r+a+b} \\
(a+r)V_u &= \overline{w} + a\left(\frac{(r+a)(w-e) + b\overline{w}}{r(r+a+b)}\right) = \frac{\overline{w}(r(r+a) + b(r+a)) + a(r+a)(w-e)}{r(r+a+b)} \\
rV_u &= \frac{\left[\overline{w}(r+b) + a(w-e)\right](a+r)}{(a+r)(r+a+b)} \\
rV_u &= \frac{\overline{w}(b+r) + a(w-e)}{(r+a+b)}
\end{aligned}$$

Substituting in the NSC condition

1.0.4 Aggregate Non Shirking Condition

In Equilibrium

$$\begin{aligned}
a(N-L) &= bL \\
a &= bL/(N-L)
\end{aligned}$$

$$\begin{aligned}
w &\geq rV_u + (r+b+q)\,e/q \\
w &\geq \frac{\overline{w}(b+r) + a(w-e)}{(r+a+b)} + (r+b+q)\,e/q
\end{aligned}$$

$$\frac{(r+b)}{r+a+b}w \geq \frac{\overline{w}\,(b+r) - ae + (r+a+b)\,(r+b+q)\,e/q}{(r+a+b)}$$

$$w \geq \overline{w} + \frac{-aqe + (r+a+b)\,(r+b+q)\,e}{(r+b)\,q} =$$

$$w \geq \overline{w} + \frac{((r+b)^2 + a(r+b) + q(r+b))\,e}{(r+b)\,q} = \overline{w} + e + \frac{(r+b+a)\,e}{q}$$

$$w \geq \overline{w} + e + \left(r + b\left(1 + \frac{L}{N-L}\right)\right)\frac{e}{q} = \overline{w} + e + \left(r + b\left(\frac{N}{N-L}\right)\right)\frac{e}{q}$$

Which gives

$$w \geq e + \overline{w} + (e/q)\,(b/u + r) \equiv \widetilde{w}$$

1.0.5 Equlibrium

$$F'\,(L) = e + \overline{w} + (e/q)\,(b/u + r)$$